

TEORIJA ELEKTRIČNIH KOLA

Tablica primarnih parametara mreže sa dva para krajeva

Tablica primarnih i sekundarnih parametara simetričnih mreža sa dva para krajeva

Parametri nekih idealnih aktivnih mreža sa dva para krajeva

Filtri

Fourier-ov red za periodične funkcije

Snage u kolima sa složenoperiodičnim eksitacijama

Fourier-ova transformacija

Osnovne osobine Fourier-ove transformacije

Parovi Fourier-ove transformacije

Osnovne osobine Laplace-ove transformacije

Traženje inverzne Laplace-ove transformacije

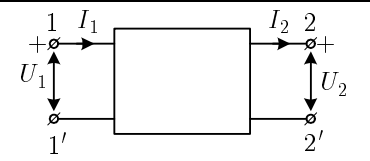
Parovi Laplace-ove transformacije

Uzimanje u obzir početnih uslova u kalemu i kondenzatoru preko ekvivalentnih šema sa nezavisnim strujnim i naponskim generatorima

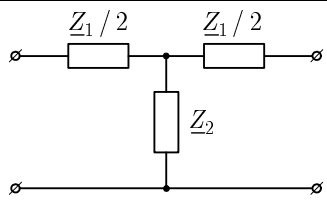
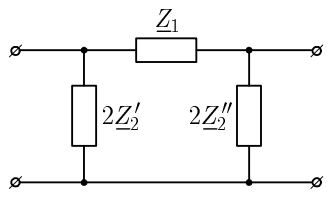
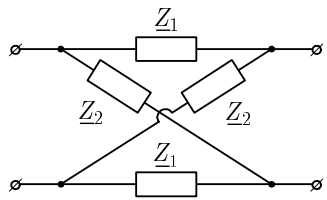
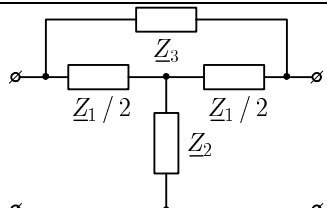
Različiti oblici zapisivanja DuHamel-ovog integrala

Neki korisni integrali

Tablica primarnih parametara mreže sa dva para krajeva

	z - parametri impedanse otvorene mreže	y - parametri admitanse kratko spojene mreže	a - parametri lančani parametri	b - parametri lančani parametri	g - parametri hibridni parametri	h - parametri hibridni parametri
$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} \underline{z}_{11} & \underline{z}_{12} \\ \underline{z}_{21} & \underline{z}_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ -I_2 \end{bmatrix}$	$\begin{matrix} \underline{z}_{11} & \underline{z}_{12} \\ \underline{z}_{21} & \underline{z}_{22} \end{matrix}$	$\begin{matrix} \frac{\underline{y}_{22}}{ \underline{y} } & -\frac{\underline{y}_{12}}{ \underline{y} } \\ -\frac{\underline{y}_{21}}{ \underline{y} } & \frac{\underline{y}_{11}}{ \underline{y} } \end{matrix}$	$\begin{matrix} \frac{\underline{a}_{11}}{\underline{a}_{21}} & \frac{1}{\underline{a}_{21}} \\ \frac{1}{\underline{a}_{21}} & \frac{\underline{a}_{22}}{\underline{a}_{21}} \end{matrix}$	$\begin{matrix} -\frac{\underline{b}_{22}}{\underline{b}_{21}} & -\frac{1}{\underline{b}_{21}} \\ -\frac{1}{\underline{b}_{21}} & -\frac{\underline{b}_{11}}{\underline{b}_{21}} \end{matrix}$	$\begin{matrix} \frac{1}{\underline{g}_{11}} & -\frac{\underline{g}_{12}}{\underline{g}_{11}} \\ \frac{\underline{g}_{21}}{\underline{g}_{11}} & \frac{ \underline{g} }{\underline{g}_{11}} \end{matrix}$	$\begin{matrix} \frac{ \underline{h} }{\underline{h}_{22}} & \frac{\underline{h}_{12}}{\underline{h}_{22}} \\ -\frac{\underline{h}_{21}}{\underline{h}_{22}} & \frac{1}{\underline{h}_{22}} \end{matrix}$
$\begin{bmatrix} I_1 \\ -I_2 \end{bmatrix} = \begin{bmatrix} \underline{y}_{11} & \underline{y}_{12} \\ \underline{y}_{21} & \underline{y}_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$	$\begin{matrix} \frac{\underline{z}_{22}}{ \underline{z} } & -\frac{\underline{z}_{12}}{ \underline{z} } \\ -\frac{\underline{z}_{21}}{ \underline{z} } & \frac{\underline{z}_{11}}{ \underline{z} } \end{matrix}$	$\begin{matrix} \underline{y}_{11} & \underline{y}_{12} \\ \underline{y}_{21} & \underline{y}_{22} \end{matrix}$	$\begin{matrix} \frac{\underline{a}_{22}}{\underline{a}_{12}} & -\frac{1}{\underline{a}_{12}} \\ -\frac{1}{\underline{a}_{12}} & \frac{\underline{a}_{11}}{\underline{a}_{12}} \end{matrix}$	$\begin{matrix} -\frac{\underline{b}_{11}}{\underline{b}_{12}} & \frac{1}{\underline{b}_{12}} \\ \frac{1}{\underline{b}_{12}} & -\frac{\underline{b}_{22}}{\underline{b}_{12}} \end{matrix}$	$\begin{matrix} \frac{ \underline{g} }{\underline{g}_{22}} & \frac{\underline{g}_{12}}{\underline{g}_{22}} \\ -\frac{\underline{g}_{21}}{\underline{g}_{22}} & \frac{1}{\underline{g}_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{\underline{h}_{11}} & -\frac{\underline{h}_{12}}{\underline{h}_{11}} \\ \frac{\underline{h}_{21}}{\underline{h}_{11}} & \frac{ \underline{h} }{\underline{h}_{11}} \end{matrix}$
$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \underline{a}_{11} & \underline{a}_{12} \\ \underline{a}_{21} & \underline{a}_{22} \end{bmatrix} \begin{bmatrix} U_2 \\ I_2 \end{bmatrix}$	$\begin{matrix} \frac{\underline{z}_{11}}{\underline{z}_{21}} & \frac{ \underline{z} }{\underline{z}_{21}} \\ \frac{1}{\underline{z}_{21}} & \frac{\underline{z}_{22}}{\underline{z}_{21}} \end{matrix}$	$\begin{matrix} -\frac{\underline{y}_{22}}{\underline{y}_{21}} & -\frac{1}{\underline{y}_{21}} \\ -\frac{ \underline{y} }{\underline{y}_{21}} & -\frac{\underline{y}_{11}}{\underline{y}_{21}} \end{matrix}$	$\begin{matrix} \underline{a}_{11} & \underline{a}_{12} \\ \underline{a}_{21} & \underline{a}_{22} \end{matrix}$	$\begin{matrix} \underline{b}_{22} & -\underline{b}_{12} \\ -\underline{b}_{21} & \underline{b}_{11} \end{matrix}$	$\begin{matrix} \frac{1}{\underline{g}_{21}} & \frac{\underline{g}_{22}}{\underline{g}_{21}} \\ \frac{\underline{g}_{11}}{\underline{g}_{21}} & \frac{ \underline{g} }{\underline{g}_{21}} \end{matrix}$	$\begin{matrix} -\frac{ \underline{h} }{\underline{h}_{21}} & \frac{\underline{h}_{11}}{\underline{h}_{21}} \\ -\frac{\underline{h}_{22}}{\underline{h}_{21}} & -\frac{1}{\underline{h}_{21}} \end{matrix}$
$\begin{bmatrix} U_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} \underline{b}_{11} & \underline{b}_{12} \\ \underline{b}_{21} & \underline{b}_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ I_1 \end{bmatrix}$	$\begin{matrix} \frac{\underline{z}_{22}}{\underline{z}_{12}} & -\frac{ \underline{z} }{\underline{z}_{12}} \\ -\frac{1}{ \underline{z} } & \frac{\underline{z}_{11}}{\underline{z}_{12}} \end{matrix}$	$\begin{matrix} -\frac{\underline{y}_{11}}{\underline{y}_{12}} & -\frac{1}{\underline{y}_{12}} \\ \frac{ \underline{y} }{\underline{y}_{12}} & -\frac{\underline{y}_{22}}{\underline{y}_{12}} \end{matrix}$	$\begin{matrix} \underline{a}_{22} & -\underline{a}_{12} \\ -\underline{a}_{21} & \underline{a}_{11} \end{matrix}$	$\begin{matrix} \underline{b}_{11} & \underline{b}_{12} \\ \underline{b}_{21} & \underline{b}_{22} \end{matrix}$	$\begin{matrix} -\frac{ \underline{g} }{\underline{g}_{12}} & \frac{\underline{g}_{22}}{\underline{g}_{12}} \\ \frac{\underline{g}_{11}}{\underline{g}_{12}} & -\frac{1}{\underline{g}_{12}} \end{matrix}$	$\begin{matrix} \frac{1}{\underline{h}_{12}} & -\frac{\underline{h}_{11}}{\underline{h}_{12}} \\ -\frac{\underline{h}_{22}}{\underline{h}_{12}} & \frac{ \underline{h} }{\underline{h}_{12}} \end{matrix}$
$\begin{bmatrix} I_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} \underline{g}_{11} & \underline{g}_{12} \\ \underline{g}_{21} & \underline{g}_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ -I_2 \end{bmatrix}$	$\begin{matrix} \frac{1}{\underline{z}_{11}} & -\frac{\underline{z}_{12}}{\underline{z}_{11}} \\ \frac{\underline{z}_{21}}{\underline{z}_{11}} & \frac{ \underline{z} }{\underline{z}_{11}} \end{matrix}$	$\begin{matrix} \frac{ \underline{y} }{\underline{y}_{22}} & \frac{\underline{y}_{12}}{\underline{y}_{22}} \\ \frac{\underline{y}_{21}}{\underline{y}_{22}} & \frac{1}{\underline{y}_{22}} \end{matrix}$	$\begin{matrix} \frac{\underline{a}_{21}}{\underline{a}_{11}} & -\frac{1}{\underline{a}_{11}} \\ \frac{1}{\underline{a}_{11}} & \frac{\underline{a}_{12}}{\underline{a}_{11}} \end{matrix}$	$\begin{matrix} -\frac{\underline{b}_{21}}{\underline{b}_{22}} & -\frac{1}{\underline{b}_{22}} \\ \frac{1}{\underline{b}_{22}} & -\frac{\underline{b}_{12}}{\underline{b}_{22}} \end{matrix}$	$\begin{matrix} \underline{g}_{11} & \underline{g}_{12} \\ \underline{g}_{21} & \underline{g}_{22} \end{matrix}$	$\begin{matrix} \frac{\underline{h}_{22}}{ \underline{h} } & -\frac{\underline{h}_{12}}{ \underline{h} } \\ -\frac{\underline{h}_{21}}{ \underline{h} } & \frac{\underline{h}_{11}}{ \underline{h} } \end{matrix}$
$\begin{bmatrix} U_1 \\ -I_2 \end{bmatrix} = \begin{bmatrix} \underline{h}_{11} & \underline{h}_{12} \\ \underline{h}_{21} & \underline{h}_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ U_2 \end{bmatrix}$	$\begin{matrix} \frac{ \underline{z} }{\underline{z}_{22}} & \frac{\underline{z}_{12}}{\underline{z}_{22}} \\ -\frac{\underline{z}_{21}}{\underline{z}_{22}} & \frac{1}{\underline{z}_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{\underline{y}_{11}} & -\frac{\underline{y}_{12}}{\underline{y}_{11}} \\ \frac{\underline{y}_{21}}{\underline{y}_{11}} & \frac{ \underline{y} }{\underline{y}_{11}} \end{matrix}$	$\begin{matrix} \frac{\underline{a}_{12}}{\underline{a}_{22}} & \frac{1}{\underline{a}_{22}} \\ -\frac{1}{\underline{a}_{22}} & \frac{\underline{a}_{21}}{\underline{a}_{22}} \end{matrix}$	$\begin{matrix} -\frac{\underline{b}_{12}}{\underline{b}_{11}} & \frac{1}{\underline{b}_{11}} \\ -\frac{1}{\underline{b}_{11}} & -\frac{\underline{b}_{21}}{\underline{b}_{11}} \end{matrix}$	$\begin{matrix} \frac{\underline{g}_{22}}{ \underline{g} } & -\frac{\underline{g}_{12}}{ \underline{g} } \\ -\frac{\underline{g}_{21}}{ \underline{g} } & \frac{\underline{g}_{11}}{ \underline{g} } \end{matrix}$	$\begin{matrix} \underline{h}_{11} & \underline{h}_{12} \\ \underline{h}_{21} & \underline{h}_{22} \end{matrix}$

Tablica primarnih i sekundarnih parametara simetričnih mreža sa dva para krajeva

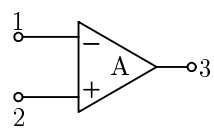
	PRIMARNI PARAMETRI			SEKUNDARNI PARAMETRI	IMPEDANSE
	a	y	z		
	$a_{11} = 1 + \frac{Z_1}{2Z_2}$ $a_{12} = Z_1 \left(1 + \frac{Z_1}{4Z_2} \right)$ $a_{21} = \frac{1}{Z_2}$ $a_{22} = 1 + \frac{Z_1}{2Z_2}$	$y_{11} = \frac{2Z_1 + 4Z_2}{Z_1(Z_1 + 4Z_2)}$ $y_{12} = -\frac{4Z_2}{Z_1(Z_1 + 4Z_2)}$ $y_{21} = \frac{4Z_2}{Z_1(Z_1 + 4Z_2)}$ $y_{22} = -\frac{2Z_1 + 4Z_2}{Z_1(Z_1 + 4Z_2)}$	$z_{11} = \frac{Z_1}{2} + Z_2$ $z_{12} = -Z_2$ $z_{21} = Z_2$ $z_{22} = -\left(\frac{Z_1}{2} + Z_2 \right)$	$Z_C = Z_C^I = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2} \right)}$ $\Gamma_C = 2 \ln \left(\sqrt{1 + \frac{Z_1}{4Z_2}} + \sqrt{\frac{Z_1}{4Z_2}} \right)$ $\operatorname{ch} \Gamma = 1 + \frac{Z_1}{2Z_2}$ $\operatorname{sh} \frac{\Gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$	$\frac{Z_1}{2} = \frac{a_{11} - 1}{a_{21}} =$ $= Z_T \frac{\operatorname{ch} \Gamma - 1}{\operatorname{sh} \Gamma} = Z_T \operatorname{th} \frac{\Gamma}{2}$ $Z_2 = \frac{1}{a_{21}} = \frac{Z_T}{\operatorname{sh} \Gamma}$
	$a_{11} = 1 + \frac{Z_1}{2Z_2}$ $a_{12} = Z_1$ $a_{21} = \frac{1}{Z_2} \left(1 + \frac{Z_1}{4Z_2} \right)$ $a_{22} = 1 + \frac{Z_1}{2Z_2}$	$y_{11} = \frac{1}{Z_1} + \frac{1}{2Z_2}$ $y_{12} = -\frac{1}{Z_1}$ $y_{21} = \frac{1}{Z_1}$ $y_{22} = -\left(\frac{1}{Z_1} + \frac{1}{2Z_2} \right)$	$z_{11} = \frac{(2Z_1 + 4Z_2) Z_2}{Z_1 + 4Z_2}$ $z_{12} = -\frac{4Z_2^2}{Z_1 + 4Z_2}$ $z_{21} = \frac{4Z_2^2}{Z_1 + 4Z_2}$ $z_{22} = -\frac{(2Z_1 + 4Z_2) Z_2}{Z_1 + 4Z_2}$	$Z_C = Z_C^{II} = \sqrt{Z_1 Z_2 \left(\frac{1}{1 + \frac{Z_1}{4Z_2}} \right)}$ $\Gamma_C = 2 \ln \left(\sqrt{1 + \frac{Z_1}{4Z_2}} + \sqrt{\frac{Z_1}{4Z_2}} \right)$ $\operatorname{ch} \Gamma = 1 + \frac{Z_1}{2Z_2}$ $\operatorname{sh} \frac{\Gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}}$	$Z_1 = a_{12} = Z_{II} \operatorname{sh} \Gamma$ $2Z_2 = \frac{a_{12}}{a_{11} - 1} =$ $= Z_{II} \frac{\operatorname{sh} \Gamma}{\operatorname{ch} \Gamma - 1} =$ $= Z_{II} \operatorname{cth} \frac{\Gamma}{2}$
	$a_{11} = \frac{Z_2 + Z_1}{Z_2 - Z_1}$ $a_{12} = \frac{2Z_2 Z_1}{Z_2 - Z_1}$ $a_{21} = \frac{2}{Z_2 - Z_1}$ $a_{22} = \frac{Z_2 + Z_1}{Z_2 - Z_1}$	$y_{11} = \frac{Z_2 + Z_1}{2Z_2 Z_1}$ $y_{12} = -\left(\frac{Z_2 - Z_1}{2Z_2 Z_1} \right)$ $y_{21} = \frac{Z_2 - Z_1}{2Z_2 Z_1}$ $y_{22} = -\left(\frac{Z_2 + Z_1}{2Z_2 Z_1} \right)$	$z_{11} = \frac{1}{2}(Z_1 + 4Z_2)$ $z_{12} = -\frac{1}{2}(Z_1 - 4Z_2)$ $z_{21} = \frac{1}{2}(Z_1 - 4Z_2)$ $z_{22} = -\frac{1}{2}(Z_1 + 4Z_2)$	$Z_C = Z_T = \sqrt{Z_1 Z_2}$ $\Gamma_C = 2 \ln \left(\sqrt{1 + \frac{Z_1}{4Z_2}} + \sqrt{\frac{Z_1}{4Z_2}} \right)$ $\operatorname{ch} \Gamma = \frac{Z_2 + Z_1}{Z_2 - Z_1}$ $\operatorname{th} \frac{\Gamma}{2} = \sqrt{\frac{Z_1}{Z_2}}$	$Z_1 = \frac{a_{11} - 1}{a_{21}} = Z_C \operatorname{th} \frac{\Gamma}{2}$ $Z_2 = \frac{a_{11} + 1}{a_{21}} = Z_C \operatorname{cth} \frac{\Gamma}{2}$ <p>ili</p> $Z_1 = \frac{a_{11} + 1}{a_{21}} = Z_C \operatorname{cth} \frac{\Gamma}{2}$ $Z_2 = \frac{a_{11} - 1}{a_{21}} = Z_C \operatorname{th} \frac{\Gamma}{2}$
				$Z_C = \sqrt{\frac{Z_1 Z_3 (Z_1 + 4Z_2)}{4(Z_1 + Z_3)}}$ $\operatorname{ch} \Gamma = 1 + \frac{Z_1}{2Z_2} \left(\frac{Z_3}{Z_2 + Z_3 + \frac{Z_1^2}{4Z_2}} \right)$	

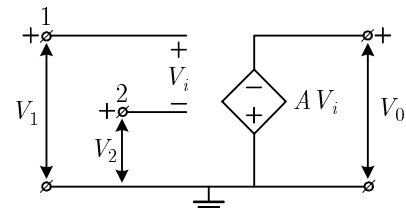
Parametri nekih idealnih aktivnih mreža sa dva para krajeva

		Naziv	Simbol	z		y		g		h		a		
KONTROLISANI IZVORI	strujom kontrolisani naponski izvor SKNI		0	0							0	0		
	naponom kontrolisani strujni izvor NKSI				0	0					0	$-\frac{1}{g}$		
	naponom kontrolisani naponski izvor NKNI						0	0			$\frac{1}{\mu}$	0		
	strujom kontrolisani strujni izvor SKSI										0	0	0	$-\frac{1}{\alpha}$
KONVERTORI	strujni negativni konvertor SNC						0	$\frac{1}{k}$	0	0	0	k	k	0
	Naponski negativni konvertor NNC						$\frac{1}{k}$	0	0	$-\frac{1}{k}$	0	-k	-k	0

	pozitivni konvertor IDEALNI TRANSFORMATOR		-----	-----	$0 \quad -\frac{1}{m}$ $\frac{1}{m} \quad 0$	$0 \quad m$ $-m \quad 0$	$m \quad 0$ $0 \quad -\frac{1}{m}$
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INVERTORI	pozitivni invertor ŽIRATOR		$0 \quad \mp r$ $\pm r \quad 0$	$0 \quad \frac{1}{\pm r}$ $\frac{1}{\mp r} \quad 0$	-----	-----	$0 \quad \pm r$ $\frac{1}{\pm r} \quad 0$
	negativni invertor		$0 \quad \mp r$ $\mp r \quad 0$	$0 \quad \frac{1}{\mp r}$ $\frac{1}{\mp r} \quad 0$	-----	-----	$0 \quad \pm r$ $\frac{1}{\mp r} \quad 0$

SIMBOL 

EKVIVALENTNA ŠEMA 

$V_o = A(V_2 - V_1) = -AV_i$

TABLICA ZAVISNOSTI IZMEĐU PRIMARNIH PARAMETARA:

$\underline{z}_{12} = \underline{z}_{21}; \underline{y}_{12} = \underline{y}_{21}; |\underline{a}| = \underline{a}_{11}\underline{a}_{22} - \underline{a}_{12}\underline{a}_{21} = 1; |\underline{b}| = \underline{b}_1\underline{b}_{22} - \underline{b}_{12}\underline{b}_{21} = 1; \underline{g}_{12} = -\underline{g}_{21}; \underline{h}_{12} = -\underline{h}_{21}$

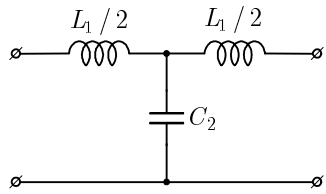
TABLICA ZAVISNOSTI IZMEĐU PRIMARNIH PARAMETARA SIMETRIČNIH MREŽ:

$\underline{z}_{11} = \underline{z}_{22}; \underline{y}_{11} = \underline{y}_{22}; \underline{a}_{11} = \underline{a}_{22}; \underline{b}_{11} = \underline{b}_{22}; |\underline{g}| = \underline{g}_{11}\underline{g}_{22} - \underline{g}_{12}\underline{g}_{21} = 1; |\underline{h}| = \underline{h}_{11}\underline{h}_{22} - \underline{h}_{12}\underline{h}_{21} = 1;$
 $|\underline{y}| = \underline{y}_{11}\underline{y}_{22} - \underline{y}_{12}\underline{y}_{21}; |\underline{z}| = \underline{z}_{11}\underline{z}_{22} - \underline{z}_{12}\underline{z}_{21}; |\underline{g}| = \underline{g}_{11}\underline{g}_{22} - \underline{g}_{12}\underline{g}_{21}; |\underline{h}| = \underline{h}_{11}\underline{h}_{22} - \underline{h}_{12}\underline{h}_{21}.$

Filtri

Filtri sa konstantnim proizvodom redne i otočne impedanse $Z_1 Z_2 = \text{const}$ nazivaju se **K-filtri**.

K-filtri niskih učestanosti:



$$Z_1 = j\omega L_1 = \omega L_1 e^{j\frac{\pi}{2}}$$

$$Z_2 = \frac{1}{j\omega C_2} = \frac{1}{\omega C_2} e^{-j\frac{\pi}{2}}$$

$$Z_1 Z_2 = \frac{L_1}{C_2} = R^2 = \text{const}$$

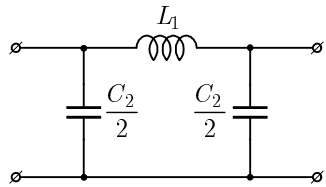
$$Z_C^T Z_C^{\text{II}} = Z_1 Z_2 = R^2$$

$$\frac{Z_1}{4Z_2} = \frac{1}{4} L_1 C_2 \omega^2 e^{j\pi} = N e^{j\pi}$$

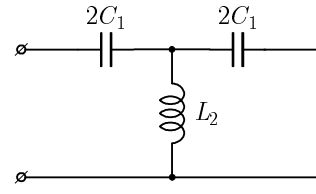
$$\Gamma_C = 2 \ln(\sqrt{1 + N e^{j\pi}} + \sqrt{N e^{j\pi}})$$

granica propusnog i nepropusnog opsega

$$N=1 \Rightarrow \omega = \omega_c = \frac{2}{\sqrt{L_1 C_2}}$$



K-filtri visokih učestanosti



$$Z_1 = \frac{1}{j\omega C_1} = \frac{1}{\omega C_1} e^{-j\frac{\pi}{2}}$$

$$Z_2 = j\omega L_2 = \omega L_2 e^{j\frac{\pi}{2}}$$

$$Z_1 Z_2 = \frac{L_2}{C_1} = R^2 = \text{const}$$

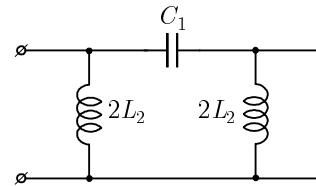
$$Z_C^T Z_C^{\text{II}} = Z_1 Z_2 = R^2$$

$$\frac{Z_1}{4Z_2} = \frac{1}{4} \frac{1}{L_2 C_1 \omega^2} e^{-j\pi} = N e^{-j\pi}$$

$$\Gamma_C = 2 \ln(\sqrt{1 + N e^{-j\pi}} + \sqrt{N e^{-j\pi}})$$

granica propusnog i nepropusnog opsega

$$N=1 \Rightarrow \omega = \omega_c = \frac{1}{2\sqrt{L_2 C_1}}$$



Campbell-ova jednačina: $\text{ch} \Gamma_C = 1 + \frac{Z_1}{4Z_2}$; $\text{ch} \Gamma_C = \text{ch}(A_C + jB_C) = \text{ch} A_C \cos B_C + j \text{sh} A_C \sin B_C$

Trigonometrijske i hiperbolne funkcije	Veza između trigonometrijskih i hiperbolnih funkcija	
$e^{jx} = \cos x + j \sin x$ $e^{-jx} = \cos x - j \sin x$ $\cos x = \frac{e^{jx} + e^{-jx}}{2}$ $\sin x = \frac{e^{jx} - e^{-jx}}{2j}$ $\cos z = \frac{e^{jz} + e^{-jz}}{2}$ $\sin z = \frac{e^{jz} - e^{-jz}}{2j}$	$\text{sh} z = -j \sin z$ $\text{sh} jz = j \sin z$ $\text{ch} z = \cos z$ $\text{ch} jz = \cos z$ $\text{th} jz = -j \tan z$ $\text{th} jz = j \tan z$ $\text{cth} z = j \text{cth} jz$ $\text{cth} jz = -j \text{cth} z$	$\arcsin z = \frac{1}{j} \log(jz \pm \sqrt{1 - z^2})$ $\arccos z = \frac{1}{j} \log(z \pm \sqrt{1 - z^2})$ $\arctan z = \frac{1}{j^2} \log \frac{j - z}{j + z}$ $\text{arccot} z = \frac{1}{j^2} \log \frac{z + j}{z - j}$
Hiperbolične funkcije	Izvodi trigonometrijskih i hiperbolnih funkcija	
$\text{ch} z = \frac{e^z + e^{-z}}{2}$ $\text{sh} z = \frac{e^z - e^{-z}}{2}$ $\text{th} z = \frac{\text{sh} z}{\text{ch} z}$ za $z \neq (2k + 1) \frac{j\pi}{2}$ ($k = 0, \pm 1, \pm 2, \dots$) $\text{cth} z = \frac{\text{ch} z}{\text{sh} z} = \frac{1}{\text{th} z}$ za $z \neq jk\pi$ ($k = 0, \pm 1, \pm 2, \dots$)	$(\sin z)' = \cos z$ $(\cos z)' = -\sin z$ $(\tan z)' = \frac{1}{\cos^2 z}$ $(\cot z)' = -\frac{1}{\sin^2 z}$ $(\text{sh} z)' = \text{ch} z$ $(\text{ch} z)' = \text{sh} z$ $(\text{th} z)' = \frac{1}{\text{ch}^2 z}$ $(\text{cth} z)' = -\frac{1}{\text{sh}^2 z}$	$\text{Arsh} z = \log(z \pm \sqrt{z^2 + 1})$ $\text{Arch} z = \log(z \pm \sqrt{z^2 - 1})$ $\text{Arth} z = \frac{1}{2} \log \frac{1 + z}{1 - z}$ $\text{Arcth} z = \frac{1}{2} \log \frac{z + 1}{z - 1}$

Fourier-ov red za periodične funkcije

Složeneriodična funkcija $f(t) = f(t + T)$ može se predstaviti Fourier-ovim redom koji predstavlja sumu prosotperiodičnih funkcija u obliku: $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$:

$$a_0 = \frac{2}{T} \int_{t_0}^{t_0+T} f(t) dt; \quad a_n = \frac{2}{T} \int_{t_0}^{t_0+T} f(t) \cos n\omega_0 t dt; \quad n = 0, 1, 2, \dots$$

gdje je:

$$b_n = \frac{2}{T} \int_{t_0}^{t_0+T} f(t) \sin n\omega_0 t dt; \quad n = 1, 2, 3, \dots$$

1. Ako je funkcija $f(t)$ parna funkcija tj. $f(-t) = f(t)$ Fourier-ov red će sadržati konstantni član i

kosinusne članove: $a_n = \frac{4}{T} \int_0^{T/2} f(t) \cos n\omega_0 t dt, \quad n = 0, 1, 2, \dots$

$$b_n = 0, \quad n = 1, 2, 3, \dots$$

2. Ako je funkcija $f(t)$ neparna funkcija tj. $f(-t) = -f(t)$ Fourier-ov red će sadržati samo sinusne

članove: $b_n = \frac{4}{T} \int_0^{T/2} f(t) \sin n\omega_0 t dt, \quad n = 1, 2, 3, \dots$

$$a_n = 0, \quad n = 0, 1, 2, \dots$$

3. Ako je negativni talas vremenske funkcije ogledalska slika pozitivnog talasa tj. ako je

$$a_n = 0, \quad \text{za } n\text{-parno}$$

$f(t + \frac{T}{2}) = -f(t)$ onda: $a_n = \frac{4}{T} \int_0^{T/2} f(t) \cos n\omega_0 t dt, \quad n\text{-neparno}$

$$b_n = 0, \quad n\text{-parno}$$

$$b_n = \frac{4}{T} \int_0^{T/2} f(t) \sin n\omega_0 t dt, \quad n\text{-neparno}$$

4. Ako funkcija $f(t)$ ispunjava uslov iz trećeg slučaja i uz to je simetrična u odnosu na koordinatni početak, tj. neparna, njen Fourier-ov red će imati samo neparne sinusne članove:

$$b_{2n+1} = \frac{8}{T} \int_0^{T/4} f(t) \sin(2n+1)\omega_0 t dt, \quad n = 1, 2, 3, \dots$$

Drugi oblici trigonometrijskog Fourier-ovog reda su: $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t + \theta_n)$;

$$a_n \cos n\omega_0 t + b_n \sin n\omega_0 t = A_n \cos(n\omega_0 t + \theta_n); \quad A_n = \sqrt{a_n^2 + b_n^2}; \quad \theta_n = -\arctan \frac{b_n}{a_n};$$

$$\text{ili } f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} A_n \sin(n\omega_0 t + \theta'_n); \quad \theta'_n = \theta_n + \frac{\pi}{2}$$

Kompleksni oblik Fourier-ovog reda.

Polazeći od relacija: $\cos n\omega_0 t = \frac{1}{2}(e^{jn\omega_0 t} + e^{-jn\omega_0 t})$ i $\sin n\omega_0 t = \frac{1}{2j}(e^{jn\omega_0 t} - e^{-jn\omega_0 t})$ dobijamo:

$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[\left(\frac{a_n - jb_n}{2} \right) e^{jn\omega_0 t} + \left(\frac{a_n + jb_n}{2} \right) e^{-jn\omega_0 t} \right]$. Uvodeći novi koeficijent $c_n = \frac{a_n - jb_n}{2}$ dobijamo

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) [\cos n\omega_0 t - j \sin n\omega_0 t] dt = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-jn\omega_0 t} dt; \quad c_n^* = \frac{1}{T} \int_{-T/2}^{T/2} f(t) [\cos n\omega_0 t + j \sin n\omega_0 t] dt$$

Ako n zamijenimo sa $-n$ dobijamo $c_{-n} = \frac{a_n + jb_n}{2} = c_n^*$; $\frac{a_0}{2} = \frac{1}{T} \int_{-T/2}^{T/2} f(t) dt = c_0$.

Sada možemo pisati: $f(t) = c_0 + \sum_{n=1}^{\infty} c_n e^{jn\omega_0 t} + \sum_{n=1}^{\infty} c_{-n} e^{-jn\omega_0 t}$. Pošto je $c_0 = c_n|_{n=0}$ dobijamo:

$f(t) = \sum_{n=0}^{\infty} c_n e^{jn\omega_0 t} + \sum_{n=-1}^{-\infty} c_n e^{jn\omega_0 t}$ pa konačno dobijamo kompleksni oblik Fourier-ovog reda:

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}; \quad c_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-jn\omega_0 t} dt$$

$$\frac{1}{T} \int_{-T/2}^{T/2} f^2(t) dt = \left(\frac{a_0}{2}\right)^2 + \sum_{n=1}^{\infty} \frac{A_n^2}{2} = \left(\frac{a_0}{2}\right)^2 + \sum_{n=1}^{\infty} \frac{a_n^2 + b_n^2}{2} \text{ -PARSERVALOVA JEDNAČINA}$$

Snage u kolima sa složenoperiodičnim eksitacijama

Aktivna snaga: $P = \sum_k P_k = \sum_k U^{(k)} I^{(k)} \cos \varphi^{(k)}$

Reaktivna snaga: $Q = \sum_k Q_k = \sum_k U^{(k)} I^{(k)} \sin \varphi^{(k)}$

Prividna snaga: $S = UI; \quad S^2 = P^2 + Q^2 + D^2$

Snaga izobličenja: $D = \sqrt{\sum_{\substack{k=0; l=0 \\ k \neq l}}^{\infty} \left[(U^{(k)})^2 (I^{(l)})^2 - 2U^{(k)} U^{(l)} I^{(k)} I^{(l)} \cos(\varphi^{(k)} - \varphi^{(l)}) + (U^{(l)})^2 (I^{(k)})^2 \right]}$

Fourier-ova transformacija

Aperiodične funkcije koje zadovoljavaju uslov: $\int_{-\infty}^{+\infty} |f(t)| dt < \infty$ imaju Fourier-ovu transformaciju:

$$F(j\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt.$$

Spektralna funkcija $F(j\omega)$ ima: moduo - $|F(j\omega)| = F(\omega); \quad F(\omega) = F(-\omega)$ (amplitudski spektar)

argument - $\arg F(j\omega) = \Psi(\omega); \quad \Psi(\omega) = -\Psi(-\omega)$ (fazni spektar)

Inverzna Fourier-ova transformacija:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(j\omega) e^{j\omega t} d\omega$$

Osnovne osobine Fourier-ove transformacije

Osobina linearnosti	$F\{c_1 f_1(t) + c_2 f_2(t)\} = c_1 F_1(j\omega) + c_2 F_2(j\omega)$
Teorema sklairanja	$F\left\{f\left(\frac{t}{a}\right)\right\} = a F(j\omega a) \quad a \equiv \text{const}$
Teorema kašnjenja	$F\{f(t - \tau)\} = e^{-j\omega\tau} F(j\omega)$ $F\{e^{j\omega_0 t} f(t)\} = F(j\omega - j\omega_0)$
Teorema simetričnosti	Ako imamo $F(jt)$ tada je Fourier-ova transformacija $2\pi f(-\omega)$. ako zamijenimo mjesta promjenljivim ω i t tada dobijamo teoremu simetričnosti.
Promjena znaka argumenta	$F\{f(-t)\} = F(-j\omega)$
Teorema o diferenciranju u vremenskom domenu	$F\left\{\frac{d^n}{dt^n} [f(t)]\right\} = (j\omega)^n F(j\omega); \quad n = 0, 1, 2, \dots$

Teorema o diferenciranju u kompleksnom domenu	$F\{t^n f(t)\} = (-1)^n \frac{d^n}{d(j\omega)^n} F(j\omega)$
Konvolucija dvije funkcije u vremenskom domenu	$F\{f_1(t) * f_2(t)\} = F_1(j\omega)F_2(j\omega)$
Konvolucija dvije funkcije u frekvencijskom domenu	$F\{f_1(t)f_2(t)\} = \frac{1}{2\pi} F_1(j\omega) * F_2(j\omega)$
Osobina modulacije	$F\{f(t) \cos \omega_0 t\} = \frac{1}{2}[F(j\omega + j\omega_0) + F(j\omega - j\omega_0)]$ $F\{f(t) \sin \omega_0 t\} = \frac{1}{j2}[F(j\omega - j\omega_0) - F(j\omega + j\omega_0)]$
Paservalova teorema u Fourier-ovoj transformaciji	$\int_{-\infty}^{+\infty} f_1(t)f_2^*(t)dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F_1(j\omega)F_2^*(j\omega)d\omega$ Ako je $f_1(t) = f_2(t) = f(t)$ dobijamo teoremu Releja: $\int_{-\infty}^{+\infty} f^2(t)dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F_1(j\omega) ^2 d\omega$

Parovi Fourier-ove transformacije		
	$F(j\omega)$	$f(t)$
1.	1	$\delta(t)$ - impulsna funkcija
2.	$2\pi\delta(\omega)$	1
3.	$\pi\delta(\omega) + \frac{1}{j\omega}$	$h(t)$ - jedinična funkcija
4.	$\frac{2}{j\omega}$	sgnt
5.	$-j\pi[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	$\sin \omega_0 t$
6.	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$\cos \omega_0 t$
7.	$\frac{\pi}{2}[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)] + \frac{j\omega}{\omega_0^2 - \omega^2}$	$[\cos \omega_0 t]h(t)$
8.	$j\frac{\pi}{2}[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)] + \frac{\omega_0}{\omega_0^2 - \omega^2}$	$[\sin \omega_0 t]h(t)$
9.	$2\pi e^{a\omega}h(-\omega); \quad a > 0$	$\frac{1}{a + jt}$
10.	$2\pi A \quad \omega < \frac{a}{2}$ $0 \quad \omega > \frac{a}{2}$	$\frac{2A}{t} \sin \frac{at}{2}$
11.	$\frac{2a}{\omega^2 + a^2}$	$e^{-at}; \quad a > 0$
12.	$\pi e^{- \omega }$	$\frac{1}{t^2 + 1}$
13.	$2\pi e^{-a \omega }$	$\frac{1}{a^2 + t^2}$
14.	$\frac{1}{a + j\omega}$	$e^{-at}h(t); \quad a > 0$
15.	$e^{-j\omega a}$	$\delta(t - a)$
16.	$\frac{n!}{(a + j\omega)^{n+1}}$	$t^n e^{-at}h(t)$
17.	$2\pi\delta(\omega - a)$	e^{-jat}
18.	$\frac{2 \sin a\omega}{\omega}$	$p_a(t)$

19.	$p_a(\omega)$	$\frac{\sin at}{\pi t}$
20.	$\frac{1}{\pi} \frac{\sin(\omega - 2)}{\omega - 2}$	$\frac{e^{j2t}}{2\pi}$, $-1 < t < 1$; 0 – van tog intervala
21.	$\frac{a + j\omega}{(a + j\omega)^2 + b^2}$	$e^{-at} \cos(bt) h(t)$
22.	$\frac{b}{(a + j\omega)^2 + b^2}$	$e^{-at} \sin(bt) h(t)$
23.	$\frac{1 - e^{-2(a+j\omega)}}{a + j\omega}$	$e^{-at} [h(t) - h(t - 2)]$
24.	$\frac{F(j\omega)}{j\omega} + \pi F(0)\delta(\omega)$	$\int_{-\infty}^t f(t) dt$
25.	$\frac{2(a^2 - \omega^2)}{(a^2 + \omega^2)^2}$	$ t e^{- at }$

Osnovne osobine Laplace-ove transformacije

Laplace-ova transformacija	$L\{f(t)\} = F(s) = \int_0^{\infty} f(t)e^{-st} dt$
Inverzna Laplace-ova transformacija	$f(t) = L^{-1}\{F(s)\} = \frac{1}{2\pi j} \int_{C_0 - j\infty}^{C_0 + j\infty} F(s)e^{st} ds = \sum \text{Res} F(s)e^{st}$
Diferenciranje originala	$L\left\{\frac{df}{dt}\right\} = sF(s) - f(0)$ $L\left\{\frac{d^n f}{dt^n}\right\} = s^n F(s) - \sum_{k=1}^n f^{(k-1)}(0)s^{(k-1)}$
Integracija originala	$L\left\{\int_0^t f(t) dt\right\} = \frac{F(s)}{s}$
Diferenciranje transformacije (diferenciranje u kompleksnom domenu)	$\frac{dF(s)}{ds} = L\{-tf(t)\}$ $\frac{d^n F(s)}{ds^n} = L\{(-t)^n f(t)\}$
Integracija transformacije (integracija u kompleksnom domenu)	$\int_s^{\infty} F(s) ds = L\left\{\frac{1}{t} f(t)\right\}$
Teorema sličnosti	$L\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$
Teorema kašnjenja	$L\{f(t - \tau)\} = e^{-s\tau} F(s)$
Teorema pomjeraja	$F(s - a) = L\{e^{at} f(t)\}$
Granične vrijednosti	$f(0) = \lim_{s \rightarrow \infty} sF(s)$ $f(\infty) = \lim_{s \rightarrow 0} sF(s)$
Teorema konvolucije u vremenskom domenu	$F_1(s)F_2(s) = L\left\{\int_0^t f_1(\tau)f_2(t - \tau) d\tau\right\} = L\left\{\int_0^t f_2(\tau)f_1(t - \tau) d\tau\right\}$
DuHamel-ov integral	$sF_1(s)F_2(s) = L\left\{\frac{d}{dt} \int_0^t f_1(\tau)f_2(t - \tau) d\tau\right\} = L\left\{\frac{d}{dt} \int_0^t f_2(\tau)f_1(t - \tau) d\tau\right\}$

Teorema konvolucije u kompleksnom domenu	$L\{f(t)g(t)\} = \frac{1}{2\pi j} F(s) * G(s)$ $F(s) * G(s) = \int_{\sigma-j\omega}^{\sigma+j\omega} F(z)G(s-z)dz = \int_{\sigma-j\omega}^{\sigma+j\omega} F(s-z)G(z)dz$
Teorema razlaganja (prosti korijeni $F_2(s) = 0$)	$L^{-1}\left\{\frac{F_1(s)}{F_2(s)}\right\} = \sum_{k=1}^n \frac{F_1(s_k)}{F_2'(s_k)} e^{s_k t}$

Traženje inverzne Laplace-ove transformacije

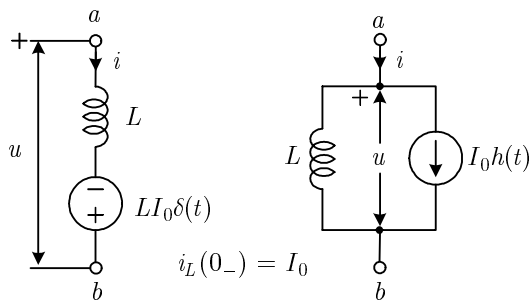
Inverzna Laplace-ova transformacija	$f(t) = \frac{1}{2\pi j} \oint e^{st} F(s) ds = \sum \text{Res} [F(s)e^{st}]$
Teorema razlaganja	$\frac{F_1(s)}{F_2(s)} = \frac{a_0 s^m + a_1 s^{m-1} + \dots + a_m}{b_0 s^n + b_1 s^{n-1} + \dots + b_n} \quad m < n$
Za slučaj kada $F_2(s) = 0$ ima proste korjene $F_2'(s_k) = \left. \frac{dF_2(s)}{ds} \right _{s=s_k}$	$L^{-1}\left\{\frac{F_1(s)}{F_2(s)}\right\} = \sum_{k=1}^n \frac{F_1(s_k)}{F_2'(s_k)} e^{s_k t}$
Za slučaj kada $F_3(s) = 0$ ima proste korjene i nema nula korjena	$L^{-1}\left\{\frac{F_1(s)}{sF_3(s)}\right\} = \frac{F_1(0)}{F_3(0)} + \sum_k^n \frac{F_1(s_k) e^{s_k t}}{s_k F_3'(s_k)}$
Za slučaj višestrukih korjena $F_2(s) = 0$	$L^{-1}\left\{\frac{F_1(s)}{F_2(s)}\right\} = \sum_{k=1}^n \frac{1}{(m_k - 1)!} \left[\frac{d^{m_k-1}}{dt^{m_k-1}} \frac{(s - s_k)^{m_k} F_1(s)}{F_2(s)} e^{s_k t} \right]_{s=s_k}$
Za slučaj kada $F_2(s) = 0$ ima dva konjugovano kompleksna korjena s i s^*	$L^{-1}\left\{\frac{F_1(s)}{F_2(s)}\right\} = 2 \text{Re} \frac{F_1(s)}{F_2'(s)} e^{st} = 2 \text{Re} \frac{F_1(s^*)}{F_2'(s^*)} e^{s^* t}$

Parovi Laplace-ove transformacije

	$F(s)$	$f(t); f(t) = 0$ za $t < 0$
1.	1	$\delta(t)$ - impulsna funkcija
2.	$\frac{1}{s}$	$h(t)$ - jedinična funkcija
3.	$\frac{1}{s^2}$	t - usponska funkcija $r(t)$
4.	$\frac{1}{s^n}$	$\frac{t^{n-1}}{(n-1)!}; n$ je cio broj
5.	$\frac{1}{s+a}$	e^{-at}
6.	$\frac{1}{(s+a)^2}$	te^{-at}
7.	$\frac{s}{(s+a)^2}$	$e^{-at}(1-at)$
8.	$\frac{1}{s(s+a)}$	$\frac{1}{a}(1-e^{-at})$
9.	$\frac{1}{s(s+a)^2}$	$\frac{1}{a^2}[1-e^{-at}(1+at)]$
10.	$\frac{1}{s^2(s+a)}$	$\frac{1}{a^2}[at - (1-e^{-at})]$
11.	$\frac{1}{(s+a)(s+b)} \quad a \neq b$	$\frac{1}{(a-b)}[e^{-at} - e^{-bt}]$

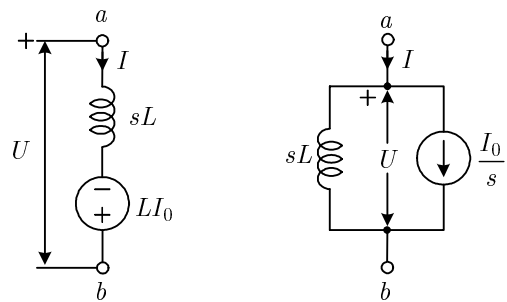
12.	$\frac{s}{(s+a)(s+b)} \quad a \neq b$	$\frac{1}{(a-b)}[ae^{-at} - be^{-bt}]$
13.	$\frac{1}{s(s+a)(s+b)} \quad a \neq b$	$\frac{1}{ab(a-b)}[be^{-at} - ae^{-bt}] + \frac{1}{ab}$
14.	$\frac{\omega}{s^2 + \omega^2}$	$\sin \omega t$
15.	$\frac{s}{s^2 + \omega^2}$	$\cos \omega t$
16.	$\frac{1}{s(s^2 + \omega^2)}$	$\frac{1}{\omega^2}[1 - \cos \omega t]$
17.	$\frac{1}{s^2 - \beta^2}$	$\frac{1}{\beta} \sinh \beta t$
18.	$\frac{s}{s^2 - \beta^2}$	$\cosh \beta t$
19.	$\frac{s+a}{(s+a)^2 + \omega^2}$	$e^{-at} \cos \omega t$
20.	$\frac{\omega}{(s+a)^2 + \omega^2}$	$e^{-at} \sin \omega t$
21.	$\frac{1}{s(s^2 - \beta^2)}$	$\frac{1}{\beta^2}[\cosh \beta t - 1]$
22.	$\frac{1}{(s+a)(s^2 + \omega^2)}$	$\frac{1}{\omega(a^2 + \omega^2)}[a \sin \omega t - \omega \cos \omega t + \omega e^{at}]$ ili $\frac{1}{\omega\sqrt{(a^2 + \omega^2)}}\left\{\sin(\omega t - \theta) + \sin \theta e^{at}\right\}, \quad \theta = \arctan\left(\frac{\omega}{a}\right)$
23.	$\frac{s}{(s+a)(s^2 + \omega^2)}$	$\frac{1}{(a^2 + \omega^2)}[a \cos \omega t - \omega \sin \omega t - ae^{-at}]$ ili $\frac{1}{\sqrt{(a^2 + \omega^2)}}\left\{\cos(\omega t - \theta) - \frac{a}{\omega} \sin \theta e^{at}\right\}, \quad \theta = \arctan\left(\frac{\omega}{a}\right)$
24.	$\frac{1}{s(s+a)(s^2 + \omega^2)}$	$\left(\frac{1}{a\omega^2}\right)\left\{1 - \frac{1}{(a^2 + \omega^2)}[a^2 \cos \omega t + a\omega \sin \omega t + \omega^2 e^{-at}]\right\}$ ili $\left(\frac{1}{a\omega^2}\right)\left\{1 - \cos \theta \cos(\omega t - \theta) - \sin^2 \theta e^{at}\right\}, \quad \theta = \arctan\left(\frac{\omega}{a}\right)$
25.	$\frac{1}{(s+a)(s^2 - \beta^2)}$	$\frac{1}{\beta(a^2 - \beta^2)}[a \sinh \beta t - \beta \cosh \beta t + \beta e^{-at}]$
26.	$\frac{s}{(s+a)(s^2 - \beta^2)}$	$\frac{1}{(a^2 - \beta^2)}[a \cosh \beta t - \beta \sinh \beta t - ae^{-at}]$
27.	$\frac{1}{s(s+a)(s^2 - \beta^2)}$	$\frac{1}{\beta^2}\left\{\frac{1}{(a^2 - \beta^2)}[a^2 \cosh \beta t - a\beta \sinh \beta t - \beta^2 e^{-at}] - 1\right\}$
28.	$\frac{1}{s^2 + 2\alpha s + \omega_0^2}$	$\omega_0 > \alpha; \quad \frac{1}{\omega_1} e^{-\alpha t} \sin \omega_1 t, \quad \omega_1 = \sqrt{(\omega_0^2 - \alpha^2)}$ $\omega_0 < \alpha; \quad \frac{1}{\beta} e^{-\alpha t} \sinh \beta t, \quad \beta = \sqrt{(\alpha^2 - \omega_0^2)}$
29.	$\frac{s}{s^2 + 2\alpha s + \omega_0^2}$	$\omega_0 > \alpha; \quad e^{-\alpha t} \left[\cos \omega_1 t - \left(\frac{\alpha}{\omega_1}\right) \sin \omega_1 t \right], \quad \omega_1 = \sqrt{(\omega_0^2 - \alpha^2)}$ $\omega_0 < \alpha; \quad e^{-\alpha t} \left[\cosh \beta t - \left(\frac{\alpha}{\beta}\right) \sinh \beta t \right], \quad \beta = \sqrt{(\alpha^2 - \omega_0^2)}$
30.	$\frac{1}{s(s^2 + 2\alpha s + \omega_0^2)}$	$\omega_0 > \alpha; \quad \left(\frac{1}{\omega_0^2}\right)\left\{1 - e^{-\alpha t} \left[\cos \omega_1 t - \left(\frac{\alpha}{\omega_1}\right) \sin \omega_1 t \right]\right\}, \quad \omega_1 = \sqrt{(\omega_0^2 - \alpha^2)}$ $\omega_0 < \alpha; \quad \left(\frac{1}{\omega_0^2}\right)\left\{1 - e^{-\alpha t} \left[\cosh \beta t - \left(\frac{\alpha}{\beta}\right) \sinh \beta t \right]\right\}, \quad \beta = \sqrt{(\alpha^2 - \omega_0^2)}$

Uzimanje u obzir početnih uslova u kalemu i kondenzatoru preko ekvivalentnih šema sa nezavisnim strujnim i naponskim generatorima.



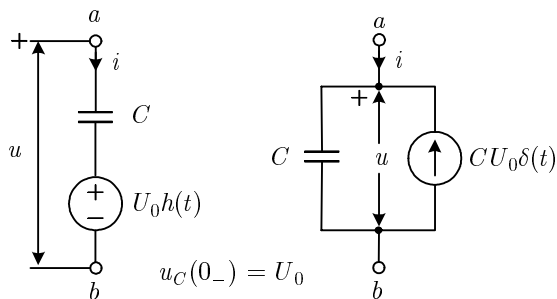
$$u(t) = -LI_0 \frac{dh(t)}{dt} + L \frac{di}{dt} = -LI_0\delta(t) + L \frac{di}{dt}$$

$$i(t) = \frac{1}{L} \int_{-\infty}^{+\infty} u(\tau) d\tau = I_0h(t) + \frac{1}{L} \int_0^t u(\tau) d\tau$$



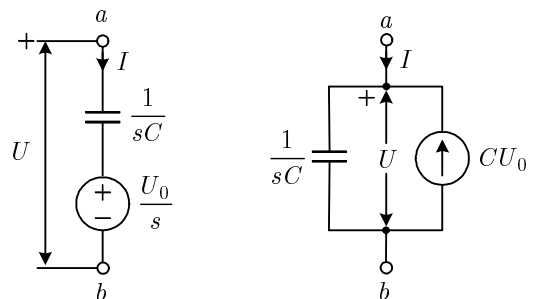
$$U = sLI - LI_0$$

$$I = \frac{U}{sL} + \frac{I_0}{s}$$



$$u(t) = \frac{1}{C} \int_{-\infty}^{+\infty} i(\tau) d\tau = U_0h(t) + \frac{1}{C} \int_0^t i(\tau) d\tau$$

$$i(t) = -CU_0\delta(t) + C \frac{du(t)}{dt}$$



$$U = \frac{I}{sC} + \frac{U_0}{s}$$

$$I = sCU - CU_0$$

Različiti oblici zapisivanja DuHamel-ovog integrala

prvi:

$$i(t) = u(0)g(t) + \int_0^t u'(\tau)g(t-\tau)d\tau$$

drugi:

$$i(t) = u(0)g(t) + \int_0^t u'(t-\tau)g(\tau)d\tau$$

treći:

$$i(t) = g(0)u(t) + \int_0^t g'(t-\tau)u(\tau)d\tau$$

četvrti:

$$i(t) = g(0)u(t) + \int_0^t g'(\tau)u(t-\tau)d\tau$$

peti:

$$i(t) = \frac{d}{dt} \int_0^t u(t-\tau)g(\tau)d\tau$$

šesti:

$$i(t) = \frac{d}{dt} \int_0^t u(\tau)g(t-\tau)d\tau$$

Neki korisni integrali

$$\int \sin^2 t \, dt = \frac{1}{2}t - \frac{1}{4}\sin 2t + C$$

$$\int \cos^2 t \, dt = \frac{1}{2}t + \frac{1}{4}\sin 2t + C$$

$$\int \frac{dt}{a^2 + t^2} = \frac{1}{a} \tan^{-1} \frac{t}{a} + C$$

$$\int te^{at} dt = \frac{1}{a^2}(at - 1)e^{at} + C$$

$$\int e^{at} \sin bt \, dt = \frac{e^{at}}{a^2 + b^2}(a \sin bt - b \cos bt) + C$$

$$\int e^{at} \cos bt \, dt = \frac{e^{at}}{a^2 + b^2}(a \cos bt + b \sin bt) + C$$

$$\int t \sin bt \, dt = \frac{1}{b^2} \sin bt - \frac{t}{b} \cos bt + C$$

$$\int t \cos bt \, dt = \frac{1}{b^2} \cos bt + \frac{t}{b} \sin bt + C$$

$$\int_0^{2\pi/\omega} \sin(\omega t + \alpha) dt = 0$$

$$\int_0^{2\pi/\omega} \cos(\omega t + \alpha) dt = 0$$

$$\int_0^{2\pi/\omega} \sin(n\omega t + \alpha) dt = 0; \quad n - \text{cio broj}$$

$$\int_0^{2\pi/\omega} \cos(n\omega t + \alpha) dt = 0; \quad n - \text{cio broj}$$

$$\int_0^{2\pi/\omega} \sin(m\omega t + \alpha) \cos(n\omega t + \alpha) dt = 0; \quad m, n - \text{cijeli brojevi}$$

$$\int_0^{2\pi/\omega} \sin^2(\omega t + \alpha) dt = \frac{\pi}{\omega}$$

$$\int_0^{2\pi/\omega} \cos^2(\omega t + \alpha) dt = \frac{\pi}{\omega}$$

$$\int_0^{2\pi/\omega} \cos(m\omega t + \alpha) \cos(n\omega t + \beta) dt = 0; \quad m \neq n, m, n - \text{cijeli brojevi}$$

$$\int_0^{2\pi/\omega} \cos(m\omega t + \alpha) \cos(n\omega t + \beta) dt = \frac{\pi \cos(\alpha - \beta)}{\omega}; \quad m = n, m, n - \text{cijeli brojevi}$$