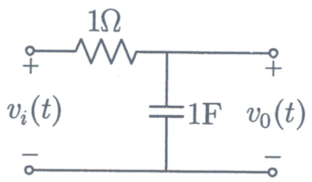
ELEKTROTEHNIČKI FAKULTET
AKADEMSKE OSNOVNE STUDIJE ELEKTROTEHNIKE 23. 11. 2006. godine
STUDIJSKI PROGRAMI: ELEKTRONIKA, RAČUNARI, TELEKOMUNIKACIJE, ENERGETIKA I AUTOMATIKA

DRUGI KOLOKVIJUM IZ TEORIJE ELEKTRIČNIH KOLA – A

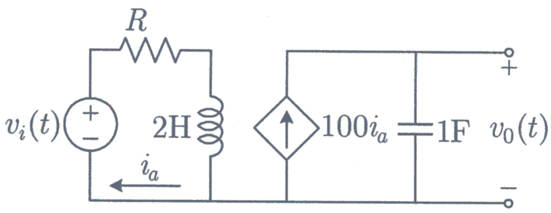
1(2p). Primjenom Laplasove transformacije odrediti odziv $v(t)$ ako je dinamika nekog električnog kola opisana sledećom relacijom $\frac{dv(t)}{dt} + 5v(t) + 6\int_0^t v(\tau)d\tau = h(t)$ i ako je $v(0) = 2$.

2(3p).



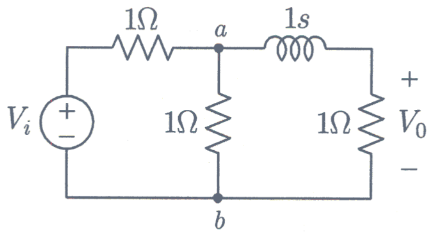
Ulazni signal u kolu prikazanom na slici je jednak $v_i(t) = 10e^{-5t}h(t)$ V. Odrediti ukupnu energiju izlaznog signala $v_0(t)$ razvijenu na otporniku od 1Ω.

3(4p).



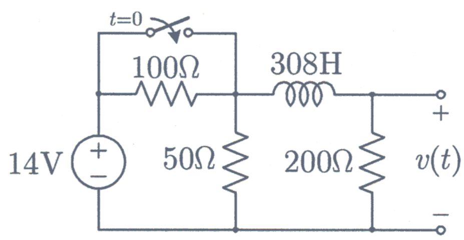
Funkcija prenosa kola prikazanog na slici jednaka je: $H(j\omega) = \frac{V_0(j\omega)}{V_i(j\omega)} = \frac{5}{j\omega(1 + j\frac{\omega}{10})}$. Odrediti vrijednost otpornosti R u prikazanom kolu u Ω.

4(6p).



Za kolo prikazano u s - domenu odrediti:
a) Funkciju prenosa kola $H(s) = V_0(s)/V_i(s)$ (1.5p).
b) Impulsni odziv kola (1.5p).
c) Jedinični (step) odziv (1.5p).
d) Odziv kada je $v_i(t) = 8\cos 2t$ V (1.5p)

5(5p).



Odziv kola prikazanog na slici je jednak $v(t) = A + Be^{-at}$ V za $t > 0$. Odrediti vrijednost konstante B u izrazu za izlazni napon u V.

NAPOMENA: Za izradu kolokvijuma student dobija tri lista i na kraju ispita moraju biti svi vraćeni. Dozvoljena je upotreba samo tabela i šema koje su dobijene na računskim vježbama. Na prvoj stranici napisati sledeće: DRUGI KOLOKVIJUM IZ TEORIJE ELEKTRIČNIH KOLA, ime i prezime i broj indeksa. Rad nastaviti na prvoj slobodnoj stranici. OBAVEZNO ISKLJUČITI MOBILNE TELEFONE.

Ispit traje 90 minuta. *PREDMETNI NASTAVNIK*

GRUPA (A)

#1 $\frac{dv(t)}{dt} + 5v(t) + 6\int_0^t v(\tau)d\tau = h(t)$; $v(0) = 2$

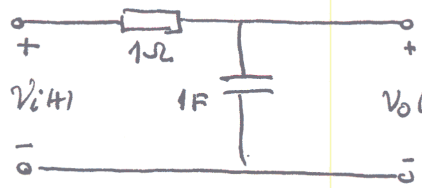
PRIMJENOM LAPLASOVE TRANSFORMACIJE NA GORNJU RELACIJU DOBIJAMO:

$[sV(s) - v(0)] + 5V(s) + \frac{6}{s}V(s) = \frac{1}{s} \Rightarrow V(s)(s^2 + 5s + 6) = 1 + 2s$

$V(s) = \frac{2s+1}{s^2+5s+6} = \frac{A}{s+2} + \frac{B}{s+3} \Rightarrow A = -3;\ B = 5$

$v(t) = \mathcal{L}^{-1}\{V(s)\} = \mathcal{L}^{-1}\left\{\frac{-3}{s+2} + \frac{5}{s+3}\right\} \Rightarrow \boxed{v(t) = (-3e^{-2t} + 5e^{-3t})}$

#2 $V_i(j\omega) = \frac{10}{j\omega+5}$; $H(j\omega) = \frac{\frac{1}{j\omega}}{1+\frac{1}{j\omega}} = \frac{1}{1+j\omega}$



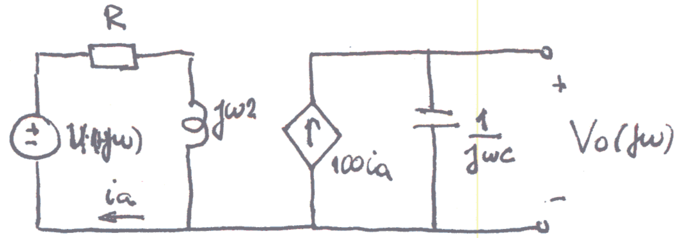
$V_o(j\omega) = H(j\omega)V_i(j\omega) = \frac{10}{(1+j\omega)(5+j\omega)}$

$|V_o(j\omega)|^2 = \frac{100}{(1+\omega^2)(25+\omega^2)} = \frac{100}{24}\left[\frac{1}{1+\omega^2} - \frac{1}{25-\omega^2}\right] = \frac{25}{6}\left[\frac{1}{1+\omega^2} - \frac{1}{25+\omega^2}\right]$

$W = \frac{1}{2\pi}\int_{-\infty}^{+\infty}|V_o(j\omega)|^2 d\omega = \frac{25}{12\pi}\left\{\int_{-\infty}^{+\infty}\frac{d\omega}{1+\omega^2} - \int_{-\infty}^{+\infty}\frac{25}{\omega^2+25}d\omega\right\}$

$W = \frac{25}{12\pi}\left\{\operatorname{arctg}(\omega)\Big|_{-\infty}^{+\infty} - \frac{1}{5}\operatorname{arctg}\left(\frac{\omega}{5}\right)\Big|_{-\infty}^{+\infty}\right\} = \frac{25}{12\pi}\left[\pi - \frac{\pi}{5}\right] = \boxed{\frac{5}{3}\ J}$

#3 (*) $H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{5}{j\omega(1+j\frac{\omega}{10})}$



SA SLIKE:

$V_o(j\omega) = \frac{1}{j\omega C}100 I_a(j\omega)$

$= \frac{1}{j\omega}100 I_a(j\omega)$

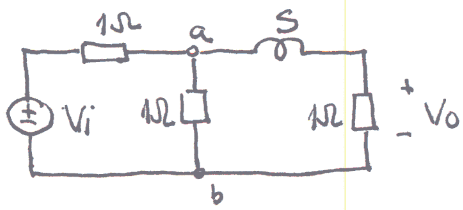
$I_a(j\omega) = \frac{V_i(j\omega)}{R+2j\omega}$

$\Rightarrow V_o(j\omega) = \frac{1}{j\omega}100\frac{V_i(j\omega)}{R+2j\omega}$

(**) $\frac{V_o(j\omega)}{V_i(j\omega)} = \frac{100}{j\omega(R+2j\omega)} = \frac{5}{j\omega(\frac{R}{20} + \frac{j\omega}{10})}$

UPOREĐUJUĆI (*) i (**) VIDIMO DA JE $\boxed{R = 20\,\Omega}$

#4 a) PRIMJENOM DJELJITELJA NAPONA DOBIJAMO:



$V_o(s) = \frac{1}{s+1}V_{ab}(s)$

$V_{ab}(s) = \frac{1 \parallel (s+1)}{1 + 1\parallel(s+1)}V_i(s) = \frac{\frac{s+1}{s+2}}{1+\frac{s+1}{s+2}}V_i(s) \Rightarrow V_{ab}(s) = \frac{s+1}{2s+3}V_i(s)$

$\boxed{H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{2s+3}}$

#4 b) $H(s) = \frac{1}{2s+3} = \frac{1}{2}\ \frac{1}{s+\frac{3}{2}}$

IMPULSNI ODZIV

$g(t) = \mathcal{L}^{-1}\{H(s)\}$ $\Rightarrow$ $\boxed{g(t) = \frac{1}{2} e^{-t\frac{3}{2}} \lambda(t)}$

c) Kada je $V_i(t) = \lambda(t)$; $V_i(s) = \frac{1}{s}$ pa je STEP odziv

$V_o(s) = H(s)V_i(s) = \frac{1}{2s(s+\frac{3}{2})} = \frac{A}{s} + \frac{B}{s+\frac{3}{2}} = \frac{1}{3}\left(\frac{1}{s} - \frac{1}{s+\frac{3}{2}}\right)$ $A = \frac{1}{3}$ $B = -\frac{1}{3}$

$V_o(t) = \mathcal{L}^{-1}\{V_o(s)\}$ $\Rightarrow$ $\boxed{V_o(t) = \frac{1}{3}(1 - e^{-\frac{3t}{2}})\lambda(t)\ V}$

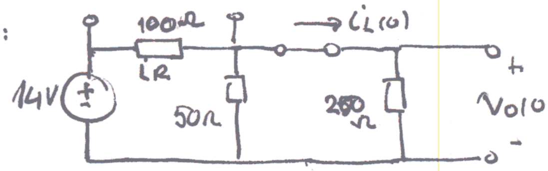
d) $V_i(t) = 8\cos 2t$ $V_i(s) = \frac{8s}{s^2+4}$

$V_o(s) = H(s)V_i(s) = \frac{4s}{(s+\frac{3}{2})(s^2+4)} = \frac{A}{s+\frac{3}{2}} + \frac{Bs+C}{s^2+4}$ | $A = -\frac{24}{25}$, $B = +\frac{24}{25}$, $C = +\frac{64}{25}$

$V_o(s) = \frac{-\frac{24}{25}}{s+\frac{3}{2}} + \frac{24}{25}\ \frac{s}{s^2+4} + \frac{32}{25}\ \frac{2}{s^2+4}$

$V_o(t) = \mathcal{L}^{-1}\{V_o(s)\}$ $\Rightarrow$ $\boxed{V_o(t) = \frac{24}{25}\left(-e^{-\frac{3}{2}t} + \cos 2t + \frac{4}{3}\sin 2t\right)\lambda(t)\ V}$

#5. KOLO U TRENUTKU $t=0$ JE OBLIKA:



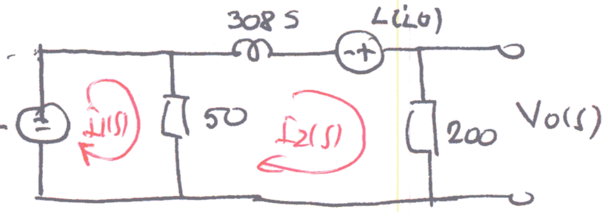
$i_R = \frac{14}{100 + \frac{50\cdot 200}{250}} = \frac{14}{140} = 0{,}1\ A$

RAZDJELNIK STRUJE:

$i_L(0) = \frac{50}{50+200}\cdot 0{,}1$ $\Rightarrow$ $\underline{i_L(0) = 0{,}02\ A}$

$L i_L(0) = 308(0{,}02) = 6{,}16\ V$

KOLO U S-DOMENU (NAKON ZATVARANJA PREKIDAČA)



$50[I_1(s) - I_2(s)] = \frac{14}{s}$

$(308s + 200)I_2(s) - Li_L(0) - 50[I_1(s) - I_2(s)] = 0$

$(308s + 200)I_2(s) = \frac{14}{s} + Li_L(0)$ $/308$

$I_2(s) = \frac{0{,}045 + 0{,}02s}{s(s+0{,}64)}$

$V_2(s) = 200\cdot I_2(s) = \frac{9+4s}{s(s+0{,}64)} = \frac{A}{s} + \frac{B}{s+0{,}64}$

$A = 14;\ B = -10$

$V_2(t) = \mathcal{L}^{-1}\{V_2(s)\} = 14 - 10e^{-0{,}64t}\ V$

$\Rightarrow$ $\boxed{B = -10\ V}$

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DRUGI KOLOKVIJUM IZ TEORIJE ELEKTRIČNIH KOLA – B

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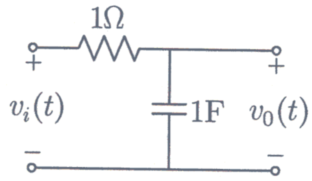
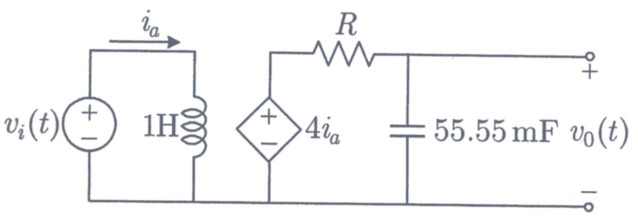
2(3p).



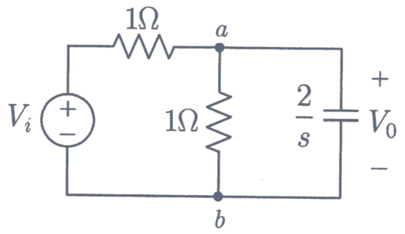
Ulazni signal u kolu prikazanom na slici je jednak $v_i(t) = 10e^{-5t}h(t)\,\mathrm{V}$. Odrediti energiju izlaznog signala $v_0(t)$ razvijenu na otporniku od 1Ω u frekventnom opsegu $2 < \omega < 4$ rad/s.

3(4p).



Funkcija prenosa kola prikazanog na slici jednaka je: $H(j\omega) = \frac{V_0(j\omega)}{V_i(j\omega)} = \frac{4}{j\omega(1 + j\frac{\omega}{18})}$. Odrediti vrijednost otpornosti R u prikazanom kolu u Ω.

4(6p).



Za kolo prikazano u s - domenu odrediti:
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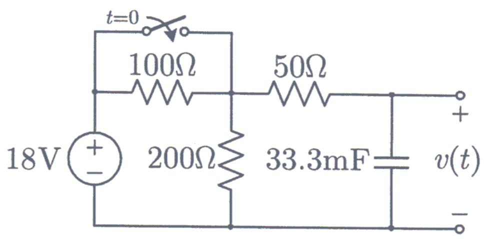
d) Odziv kada je $v_i(t) = 8\cos 2t\,\mathrm{V}$ (1.5p).

5(5p).



Odziv kola prikazanog na slici je jednak $v(t) = A + Be^{-at}\,\mathrm{V}$ za $t > 0$. Odrediti vrijednost konstante B u izrazu za izlazni napon u V.

<u>NAPOMENA:</u> Za izradu kolokvijuma student dobija <u>tri</u> lista i na kraju ispita moraju biti <u>svi</u> vraćeni. Dozvoljena je upotreba samo tabela i šema koje su dobijene na računskim vježbama. Na prvoj stranici napisati sledeće: DRUGI KOLOKVIJUM IZ TEORIJE ELEKTRIČNIH KOLA, ime i prezime i broj indeksa. Rad nastaviti na prvoj slobodnoj stranici. <u>OBAVEZNO ISKLJUČITI MOBILNE TELEFONE.</u>

Ispit traje 90 minuta.

PREDMETNI NASTAVNIK

GRUPA B

#1 $\frac{dv(t)}{dt} + 3v(t) + 2\int_0^t v(\tau)d\tau = 2e^{-3t}$; $v(0)=0$

PRIMJENOM LAPLASOVE TRANSFORMACIJE DOBIJAMO:

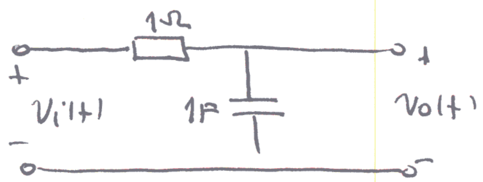
$$sV(s) - v(0) + 3V(s) + \frac{2}{s}V(s) = \frac{2}{s+3}$$

$$[s^2+3s+2]V(s) = \frac{2s}{s+3} \Rightarrow V(s) = \frac{2s}{(s+1)(s+2)(s+3)}$$

$$V(s) = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3} \{A=-1,\ B=4;\ C=-3\} = -\frac{1}{s+1} + \frac{4}{s+2} - \frac{3}{s+3}$$

$$\boxed{v(t) = \mathcal{L}^{-1}\{V(s)\} = (-e^{-t} + 4e^{-2t} - 3e^{-3t})\,h(t)\ V}$$

#2 $V_i(j\omega) = \frac{10}{5+j\omega}$; $H(j\omega) = \frac{1}{1+j\omega}$



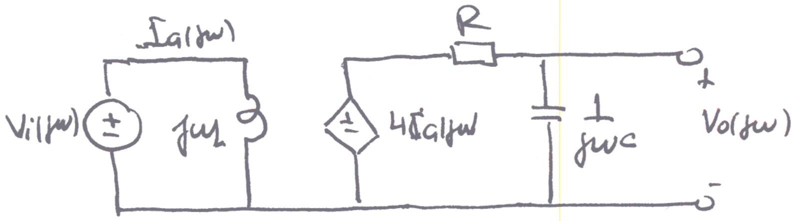
$$V_o(j\omega) = H(j\omega)V_i(j\omega) = \frac{10}{(1+j\omega)(5+j\omega)}$$

$$|V_o(j\omega)|^2 = \frac{25}{6}\left[\frac{1}{1+\omega^2} - \frac{1}{25+\omega^2}\right]$$

$$W = \frac{25}{6\pi}\left[\int_2^4 \frac{d\omega}{1+\omega^2} - \int_2^4 \frac{d\omega}{25+\omega^2}\right] = \frac{25}{6\pi}\left\{\mathrm{arctg}(\omega)\Big|_2^4 - \frac{1}{5}\mathrm{arctg}\left(\frac{\omega}{5}\right)\Big|_2^4\right\}$$

$$W = \frac{25}{6\pi}\left\{1{,}326 - 1{,}107 - \frac{(0.675-0{,}381)}{5}\right\} \Rightarrow \boxed{W = 0{,}212\ J}$$

#3 $V_i(j\omega) = I_a(j\omega)\cdot j\omega L$



$$I_a(j\omega) = \frac{V_i(j\omega)}{j\omega}$$

$$\frac{4V_i(j\omega)}{j\omega} = \frac{1+j\omega RC}{j\omega C}$$

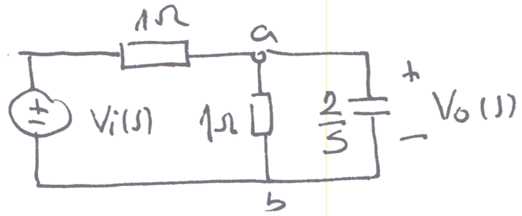
$$V_o(j\omega) = \frac{1}{j\omega C}\cdot\frac{4C\,V_i(j\omega)}{1+j\omega RC} \Rightarrow H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{4}{j\omega(1+j\omega CR)} \quad (*)$$

$$H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{4}{j\omega\left(1+j\frac{\omega}{18}\right)} \quad (**)$$

IZ (*) i (**)

$$\Rightarrow RC = \frac{1}{18} \Rightarrow R = \frac{10^3}{18\cdot 55.55} \Rightarrow \boxed{R = 1\Omega}$$

#4 a) $\frac{V_o(s)}{V_i(s)} = \frac{1 \| \frac{2}{s}}{1 + \frac{2}{s}\|1} = \frac{\frac{2s}{1+\frac{2}{s}}}{1+\frac{2s}{1+\frac{2}{s}}} = \frac{2}{s+4}$



$$\boxed{H(s) = \frac{V_o(s)}{V_i(s)} = \frac{2}{s+4}}$$

b) $H(s) = \frac{2}{s+4} \Rightarrow$ Impulsni odziv je $g(t) = \mathcal{L}^{-1}\{H(s)\}$

$$\boxed{g(t) = 2e^{-4t}h(t)}$$

#4 c) $V_o(s) = H(s)V_i(s) = \frac{2}{s(s+4)} = \frac{A}{s} + \frac{B}{s+4} \quad \begin{cases} A = \frac{1}{2} \\ B = -\frac{1}{2} \end{cases}$

$V_o(s) = \frac{1}{2}\left[\frac{1}{s} - \frac{1}{s+4}\right] \Rightarrow v_o(t) = \mathcal{L}^{-1}\{V_o(s)\}$

$$\boxed{v_o(t) = \frac{1}{2}\left(1 - e^{-4t}\right) h(t) \text{ V}}$$

d) $v_i(t) = 8\cos 2t \Rightarrow V_i(s) = \frac{8s}{s^2+4}$

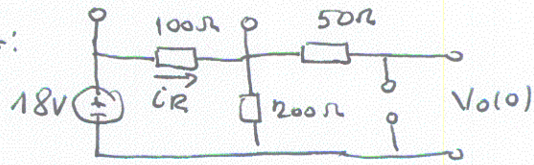
$V_o(s) = H(s)V_i(s) = \frac{16s}{(s+4)(s^2+4)} = \frac{A}{s+4} + \frac{Bs+C}{s^2+4}$

$\left\{ A = -\frac{16}{5};\ B = \frac{16}{5};\ C = \frac{16}{5} \right\} \Rightarrow V_o(s) = \frac{16}{5}\left[-\frac{1}{s+4} + \frac{s+1}{s^2+4}\right]$

$V_o(s) = \frac{16}{5}\left[-\frac{1}{s+4} + \frac{s}{s^2+4} + \frac{1}{2}\,\frac{2}{s^2+4}\right]$

$$\boxed{v_o(t) = \mathcal{L}^{-1}\{V_o(s)\} = \frac{16}{5}\left[-e^{-4t} + \cos 2t + \frac{1}{2}\sin 2t\right] h(t) \text{ V}}$$

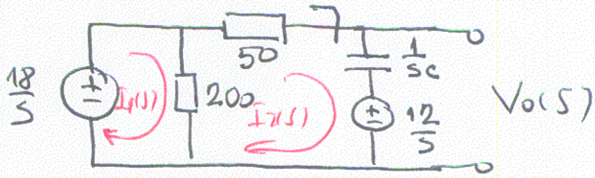
#5 KOLO U TRENUTKU $t=0$ JE OBLIKA:



$i_R = \frac{18}{100+200} = 0,06 \text{ A}$

$V_o(0) = 200 \cdot 0,06 = 12 \text{ V}$

KOLO U S-DOMENU (NAKON ZATVARANJA PREKIDAČA)



$\frac{1}{sC} = \frac{1}{s \cdot \frac{100}{3} \cdot 10^{-3}} = \frac{3}{0.1 \cdot s}$

$200\left[I_1(s) - I_2(s)\right] = \frac{18}{s}$

$I_2(s)\left(50 + \frac{3}{0.1 \cdot s}\right) + \frac{12}{s} - \frac{18}{s} = 0$

$I_2(s)\left(50 + \frac{3}{0.1 \cdot s}\right) = \frac{6}{s} \Big/ \frac{s}{50} \Rightarrow \left(s + \frac{3}{5}\right) I_2(s) = \frac{3}{25} = 0,12$

$I_2(s) = \frac{0.12}{s+0.6}$

$V_o(s) = \frac{3}{0.1 s} I_2(s) + \frac{12}{s} = \frac{3.6}{s(s+0.6)} + \frac{12}{s}$

$V_o(s) = \frac{A}{s} + \frac{B}{s+0.6} + \frac{12}{s} \left\{ \begin{matrix} A = 6 \\ B = -6 \end{matrix} \right\} = \frac{18}{s} - \frac{6}{s+0,6}$

$\left. \begin{matrix} v_o(t) = 18 - 6e^{-0.6t} \\ v_o(t) = A + Be^{-at} \end{matrix} \right\} \Rightarrow \boxed{B = -6 \text{ V}}$