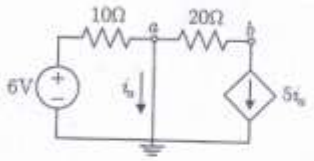
ELEKTROTEHNIČKI FAKULTET
AKADEMSKE OSNOVNE STUDIJE ELEKTROTEHNIKE 14. 10. 2006. godine
STUDIJSKI PROGRAMI: ELEKTRONIKA, RAČUNARI, TELEKOMUNIKACIJE, ENERGETIKA I AUTOMATIKA

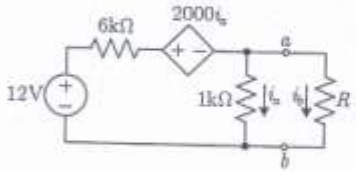
PRVI KOLOKVIJUM IZ TEORIJE ELEKTRIČNIH KOLA – A

1. (3p)



U kolu prikazanom na slici odrediti potencijale čvorova a i b.

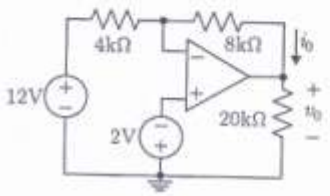
2. (3p)



U kolu prikazanom na slici odrediti vrijednost otpornosti R tako da struja i_b bude jednaka 2mA. Vrijednost struje i_a je u amperima.

Napomena: *Kolo lijevo od tačaka a i b zamijeniti Teveninovim generatorm.*

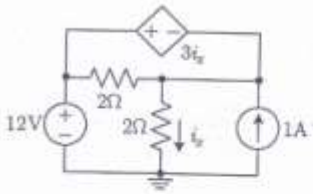
3. (3p)



Za kolo prema slici sa idealnim OP odrediti vrijednosti struje i_0 i napona v_0.

4. (3p) Neko električno kolo je za $t > 0$ opisano diferencijalnom jednačinom: $\frac{d^2v}{dt^2} + 5\frac{dv}{dt} + 6v = v_s$. Ako je $v_s = 2e^{-2t}$ V odrediti prinudni odziv u kolu $v_f = ?$ za $t > 0$.

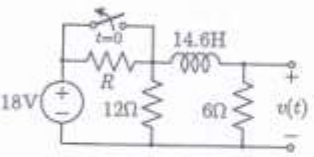
5. (4p)



Za dato električno kolo:

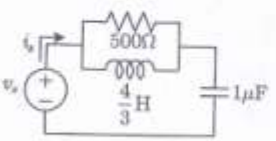
a) Nacrtati graf električnog kola i označiti osnovne presjeke. (2p)
b) Formirati matricu Q_f. (1p)
c) Odrediti struju $i_x = ?$. (1p)

6. (4p)



Odziv u kolu prikazanom na slici je jednak: $v(t) = 8 + 10e^{-0.65t}$ V za $t > 0$. Kolika je vrijednost otpornosti R u omima?

7. (6p)



Odrediti struju $i_s(t)$ za $t > 0$ u kolu prikazanom na slici ako je:

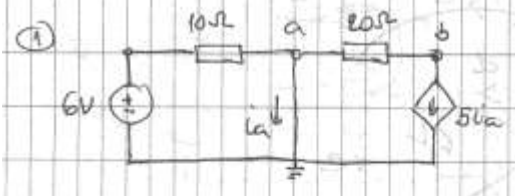
a) $v_s(t) = 10h(-t)$
b) $v_s(t) = 10h(t)$

NAPOMENA: Za izradu kolokvijuma student dobija tri lista i na kraju ispita moraju biti svi vraćeni. RADI SE UKUPNO 5 ZADATAKA. ZADACI 5, 6 i 7 SU OBAVEZNI A OD OSTALA 4 STUDENT BIRA I RADI 2 ZADATKA. Dozvoljena je upotreba samo tabela i šema koje su dobijene na računskim vježbama. Na prvoj stranici napisati sledeće: PRVI KOLOKVIJUM IZ TEORIJE ELEKTRIČNIH KOLA, ime i prezime i broj indeksa. Rad nastaviti na prvoj slobodnoj stranici. OBAVEZNO ISKLJUČITI MOBILNE TELEFONE.

Ispit traje 90 minuta.

PREDMETNI NASTAVNIK

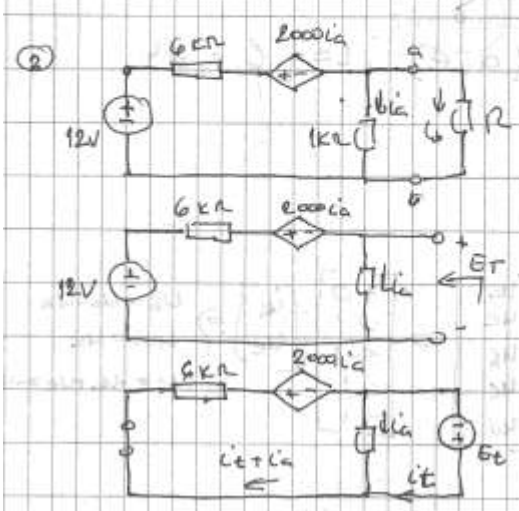
I KOLOKVIJUM GRUPA (A)

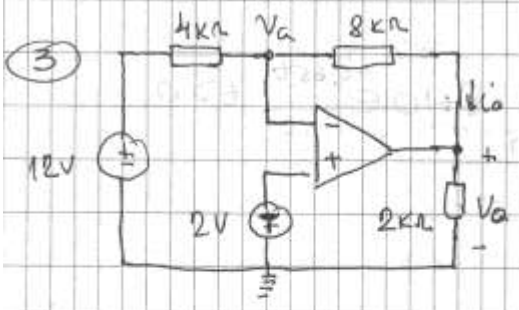
① 

$\frac{6-V_a}{10} = i_a + \frac{V_a - V_b}{20}$, $V_a = 0\ V$

$\Rightarrow i_a = \frac{12+V_b}{20}$

$\frac{0-V_b}{20} = 5i_a \Rightarrow \boxed{V_b = -10\ V}$

② 
6kΩ
2000i_a
a
i_a
1kΩ
i_b
R
12V
b
E_T
R_T
i_b
$i_b = 2\ mA$
$R = ?$
$i_b = \frac{E_T}{R_T + R}$
$-12 + 6\cdot10^3 i_a + 2000 i_a + 10^3 i_a = 0$
$\Rightarrow i_a = \frac{4}{3}\ mA$
$\boxed{E_T = \frac{4}{3}\ V}$
$6\,10^3 (i_a + i_t) + 2000 i_a = E_t$;
$E_t = -10^3 i_a \Rightarrow i_a = -E_t 10^{-3}\ A$
$\Rightarrow 6\,10^3 i_t - 6E_t - 2E_t = E_t \Rightarrow \boxed{R_T = \frac{E_t}{i_t} = 667\,\Omega}$
$i_b = \frac{\frac{4}{3}}{667+R}$; $i_b = 0002\,A \Rightarrow R = \frac{\frac{4}{3}}{0002} - 667 \Rightarrow \boxed{R = 0\,\Omega}$

③ 

Sa slike vidimo da je: $\boxed{V_a = -2V}$

Primjenom I.K.Z za čvor a imamo

$\frac{V_o - (-2)}{8000} + \frac{12-(-2)}{4000} = 0 \Rightarrow \boxed{V_o = -30V}$

$i_o = \frac{V_a - V_o}{8k\Omega} \Rightarrow \boxed{i_o = 2{,}5\ mA}$

④ $\frac{d^2V}{dt^2} + 5\frac{dV}{dt} + 6V = V_s$; $V_s = 2e^{-2t}$; $V_f = ?$ za $t > 0$

$s^2 + 5s + 6 = 0 \Rightarrow s_1 = -2$; $s_2 = -3$

$V_n = Ae^{-2t} + Be^{-3t}$ $\quad \boxed{y_p(t) = t^p K e^{\alpha t}}$

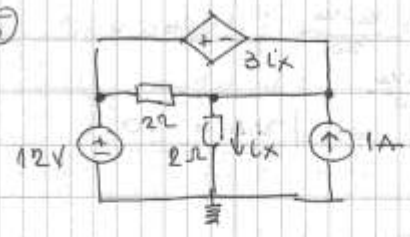
Imamo poklapanje prinudnog odziva sa komponentom sopstvenog odz.

$y_p = Kte^{-2t}$; $y_p'(t) = Ke^{-2t} - 2te^{-2t}K$; $y''(t) = -4Ke^{-2t} + 4tKe^{-2t}$

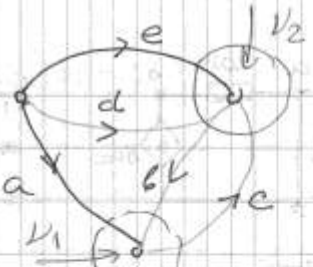
Zamjenom u polaznu d.jj. $\Rightarrow K = 2$.

$\boxed{y_p(t) = 2te^{-2t},\ t > 0}$

⑤



a)



$T = \{a, e\}$; $L = \{b, c, d\}$

b) $Q_f = \begin{array}{c} \\ V_1 \\ V_2 \end{array} \begin{array}{c} \begin{array}{ccccc} a & e & b & c & d \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & -1 & 1 & 1 \end{bmatrix} \end{array}$

$Q_f = [\, Q_{fT} \;\; Q_{fL} \,]$

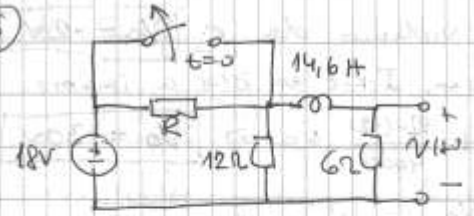
c) $i_x = i_b = ?$ $\quad u = Q_f^T u_T = \begin{bmatrix} u_a \\ u_e \\ u_b \\ u_c \\ u_d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_a \\ u_e \end{bmatrix} \Rightarrow$ $u_b = u_a - u_e$, $u_d = u_e$, $u_c = -u_a + u_e = -u_b$

$u_b = 2i_x$; $u_e = 3i_x$ $\Rightarrow u_e = \frac{3}{2} u_b \Rightarrow u_b = \frac{2}{3} u_e$

$u_b = 12 - u_e \Rightarrow \frac{2}{3} u_e = 12 - u_e \Rightarrow u_e = \frac{36}{5}\,V$

$i_x = \frac{u_e}{3} = \frac{36}{5 \cdot 3} = \frac{12}{5} = 2{,}4\,A$ $\quad \boxed{i_x = 2{,}4\,A}$

⑥

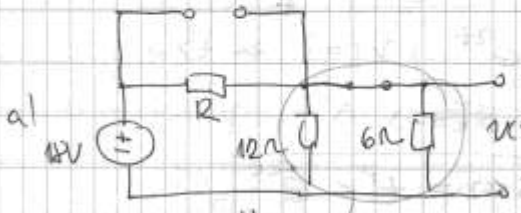


PREKIDAČ OTVOREN

$v(t) = 8 + 10e^{-0{,}65t}$, $t > 0$.

$R = ?$

a)



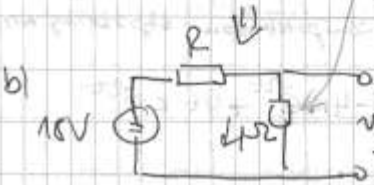
$v(\infty) = 8 + 10e^{-\infty} = 8\,V$

Razdjelnik napona na slici b) dva

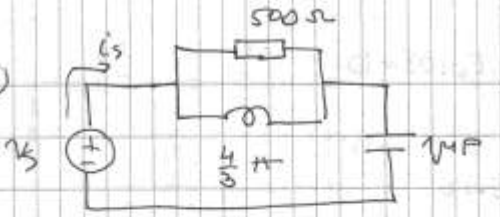
$8 = \frac{4}{R+4}(18)$

$8R + 32 = 72 \Rightarrow$ $\boxed{R = 5\,\Omega}$

b)



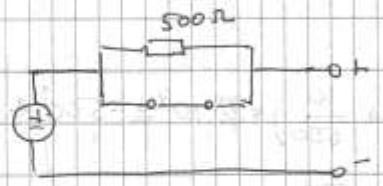
④



a) $v_s(t) = 10\,\mathcal{U}(-t)$

b) $v_s(t) = 10\,\mathcal{U}(t)$

a) $t = 0$

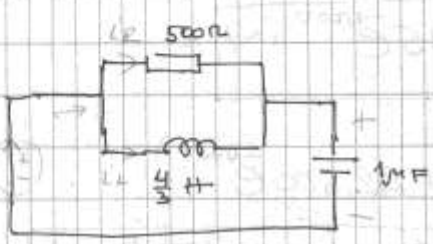


$i_L(0-) = 0$ A

$v_c(0-) = 10$ V

$i_L(0+) = i_L(0-) = 0$ A ; $v_c(0+) = v_c(0-) = 10$ V

$t > 0$



$i_s = i_L + i_R$

$C\frac{dv_c}{dt} = -\frac{v_c}{R} - \frac{1}{L}\int v_c\,dt \;/\frac{d}{dt}$

$C\frac{d^2v_c}{dt^2} + \frac{dv_c}{dt}\frac{1}{R} + \frac{1}{L}v_c = 0$

$\frac{d^2v_c}{dt^2} + \frac{1}{RC}\frac{dv_c}{dt} + \frac{1}{LC}v_c = 0$

$\frac{d^2v_c}{dt^2} + 2000\frac{dv_c}{dt} + 750000\,v_c = 0$

$s^2 + 2000s + 750000 = 0 \Rightarrow s_1 = -500$; $s_2 = -1500$

$v_c(t) = A\,e^{-500t} + B\,e^{-1500t}$

$v_c(0+) = 10 = A + B$

$C v_c'(0+) = i_R(0+) + i_L(0+)$

$v_c'(0+) = -\frac{v_c(0+)}{RC} + \frac{i_L(0+)}{C}^{\,0} \Rightarrow \frac{dv_c}{dt} = -\frac{v_c(0)}{RC} = -20000$

$\left.\begin{array}{l} A+B=0 \\ 5A+15B=200 \end{array}\right\} \Rightarrow A = -5\,,\ B = 15$

$$\boxed{v_c(t) = -5e^{-500t} + 15e^{-1500t}}\ \text{V}\,;\quad i_s = i_C = Cv_c'$$

$$\boxed{i_s = 10^{-6}(2500e^{-500t} - 22{,}1500e^{-1500t}) = 2{,}5e^{-500t} - 22{,}5e^{-1500t}\ \text{mA}}$$

b) $v_s(t) = 10\,k(t)$ V, $v_c(0) = 0$; $i_L(0) = 0$

$v_{cf} = 10$ V

$v_c(t) = 10 + A e^{-500t} + B e^{-1500t}$

$A + B = -10$

$v_c'(0+) = 10^6 i_c(0+) = 10^6 \left(0 + \frac{10}{500}\right) = 2 \cdot 10^4 = -500A - 1500B$

$\Rightarrow A = 5$; $B = -15$

$$\boxed{v_c(t) = 10 + 5e^{-500t} - 15e^{-1500t} \text{ V}}$$

$i_s = i_c = 10^{-6}\left(-2500e^{-500t} + 22{,}500e^{-1500t}\right)$

$$\boxed{i_s(t) = 25e^{-500t} + 22{,}5e^{-1500t} \text{ mA}}\ ;\ t > 0$$

$[10 = u + u_c \Rightarrow u = 10 - u_c]$

b) $C\frac{du_c}{dt} = i_s = i_L + i_R = \frac{1}{L}\int (10 - u_c)\,dt + \frac{10 - u_c}{R}$

$C\frac{du_c}{dt} = \frac{1}{L}\int 10\,dt - \frac{1}{L}\int u_c\,dt + \frac{10}{R} - \frac{u_c}{R} \;/\; \frac{d}{dt}$

$C\frac{d^2u_c}{dt^2} = \frac{10}{L} - \frac{1}{L}u_c + 0 - \frac{du_c}{R\,dt}$

$\frac{d^2u_c}{dt^2} + \frac{du_c}{dt}\frac{1}{RC} + \frac{1}{LC}u_c = \frac{10}{LC}$!!!

$u_{cf} = A \Rightarrow \frac{A}{LC} = \frac{10}{LC} \Rightarrow A = 10$

$$\boxed{u_{cf} = 10 \text{ V}}$$

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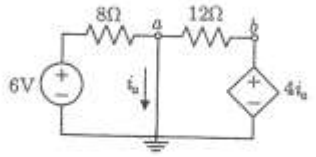
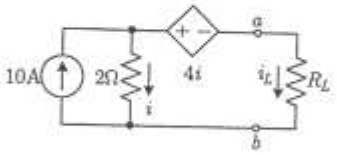
PRVI KOLOKVIJUM IZ TEORIJE ELEKTRIČNIH KOLA – B

1. (3p)



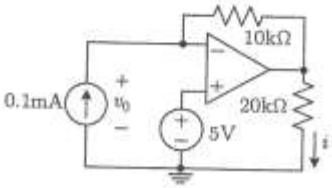
U kolu prikazanom na slici odrediti potencijale čvorova a i b.

2. (3p)



U kolu prikazanom na slici odrediti vrijednost otpornosti R_L tako da struja i_L bude jednaka -2A. **Napomena**: *Kolo lijevo od tačaka a i b zamijeniti Teveninovim generatorom.*

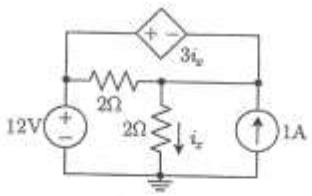
3. (3p)



Za kolo prema slici sa idealnim OP odrediti vrijednosti struje i i napona v_0.

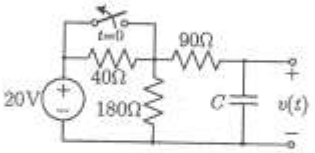
4. (3p) Neko električno kolo je za $t > 0$ opisano diferencijalnom jednačinom: $\frac{d^2i}{dt^2} + 5\frac{di}{dt} + 20i = 6i_s$. Ako je $i_s = 6 + 2t$ A odrediti prinudni odziv u kolu $i_f = ?$ za $t > 0$.

5. (4p)



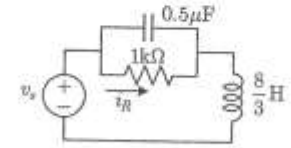
Za dato električno kolo:
a) Nacrtati graf električnog kola i označiti osnovne konture. (2p)
b) Formirati matricu B_f. (1p)
c) Odrediti struju $i_x = ?$. (1p)

6. (4p)



Odziv u kolu prikazanom na slici je jednak: $v(t) = 12 + 8e^{-0.75t}$ V za $t > 0$. Kolika je kapacitivnost C kondenzatora na slici u mF ?

7. (6p)



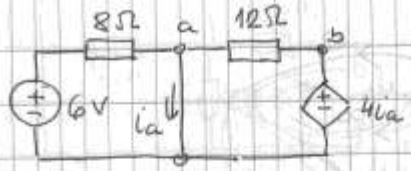
Odrediti struju $i_R(t)$ za $t > 0$ u kolu prikazanom na slici ako je:
a) $v_s(t) = 10h(-t)$
b) $v_s(t) = 10h(t)$

NAPOMENA: Za izradu kolokvijuma student dobija tri lista i na kraju ispita moraju biti svi vraćeni. RADI SE UKUPNO 5 ZADATAKA. ZADACI 5, 6 i 7 SU OBAVEZNI A OD OSTALA 4 STUDENT BIRA I RADI 2 ZADATKA. Dozvoljena je upotreba samo tabela i šema koje su dobijene na računskim vježbama. Na prvoj stranici napisati sledeće: PRVI KOLOKVIJUM IZ TEORIJE ELEKTRIČNIH KOLA, ime i prezime i broj indeksa. Rad nastaviti na prvoj slobodnoj stranici. OBAVEZNO ISKLJUČITI MOBILNE TELEFONE.

Ispit traje 90 minuta.

PREDMETNI NASTAVNIK

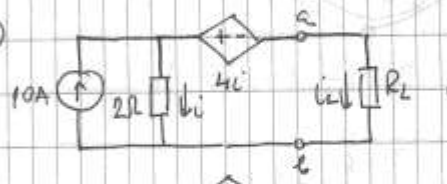
I KOLOKVIJUM – GRUPA B

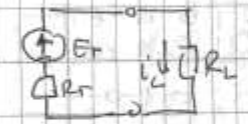
① 

I.K.Z. za čvor a

$$\frac{6-v_a}{8} = i_a + \frac{v_a - v_b}{12} \Rightarrow \boxed{i_a = \frac{6}{8} + \frac{v_b}{12}}$$

$$v_b = 4 i_a \Rightarrow \boxed{v_b = 4{,}5\,V} \quad \boxed{v_a = 0}$$

② 



$i_L = -2A$

$R_L = ?$

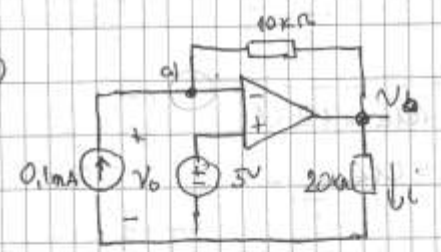
$$i_L = \frac{E_T}{R_T + R} = -2$$

$$10 = i + 0 \Rightarrow i = 10A$$

$$E_T + 4i - 2i = 0 \Rightarrow \boxed{E_T = -20V}$$

$$-V_t - 4I_t + 2I_t \Rightarrow \boxed{R_T = \frac{V_t}{I_t} = -2\Omega}$$

$$-2 = i_L = \frac{-20}{R_L - 2} \Rightarrow \boxed{R_L = 12\,\Omega}$$

③ 

Sa slike vidimo da je:

$\boxed{v_0 = 5V}$, I.K.Z. za čvor ⓐ

$$-\left(\frac{v_b - 5}{10000}\right) - 0{,}1\cdot 10^{-3} - 0 = 0 \Rightarrow \boxed{v_b = 4V}$$

$$i = \frac{v_b}{20000} \Rightarrow \boxed{i = \frac{1}{5}\,mA}$$

④ $\frac{d^2i}{dt^2} + 5\frac{di}{dt} + 20i = 6i_s$; $i_s = 6 + 2t\,A$; $i_f = ?$ $t > 0$

$s^2 + 5s + 20 = 0$

$i_{fp}(t) = A + Bt = i_f(t)$

Zamjenom u d.-lj. jednačinu imamo:

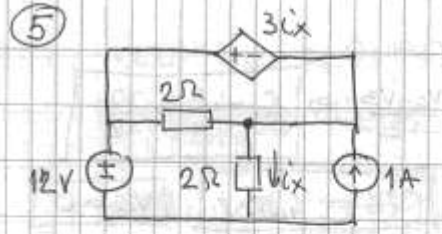
$$0 + 9B + 20(A + Bt) = 36 + 12t$$

$$9B + 2A = 36$$

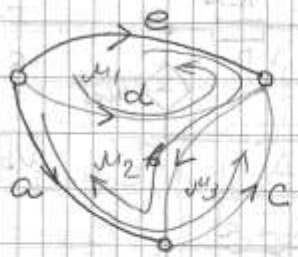
$$20B = 12 \Rightarrow \boxed{B = \frac{12}{20} = 0{,}6}; \quad \boxed{A = 1{,}53}$$

$$\boxed{i_f(t) = 1{,}53 + 0{,}6t}$$

⑤



a)



b) $B_f = \begin{matrix} \\ M_1 \\ M_2 \\ M_3 \end{matrix} \begin{matrix} \begin{bmatrix} a & e & b & d & e \\ -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$

$i^\circ = B_f^t \, i_L \Rightarrow \begin{bmatrix} i_a \\ i_e \\ i_b \\ i_d \\ i_c \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_b \\ i_c \\ i_a \end{bmatrix} \Rightarrow \begin{matrix} i_a = -i_b + i_d \\ i_e = i_b - i_c - i_d \end{matrix}$

$i_x = ?$ $\quad i_x = i_b$; $i_c = 1A$

$i_a = -i_b + i_d$

$i_e = i_b - 1 - i_d$

M_1: $\sum u = 0 \quad u_e + u_b - u_c = 0$; $u_c = 12V$

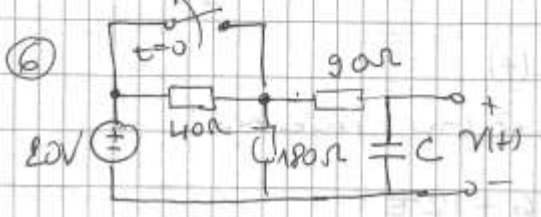
M_2 $\sum u = 0 \quad u_d - u_e = 0 \Rightarrow u_d = u_e$

$u_b = 2i_b$; $u_e = 3i_b$ $\Rightarrow u_b = \frac{2}{3} u_e$

$u_e + u_b = 12 \Rightarrow u_e + \frac{2}{5} u_e = 12 \Rightarrow \boxed{u_e = \frac{36}{5}}$

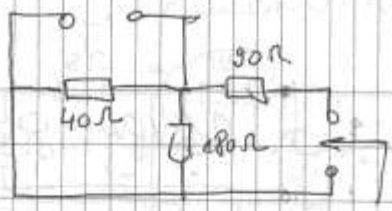
$u_e = 3i_x \Rightarrow i_x = \frac{12}{5}$ $\quad \boxed{i_x = 2,4\,A}$

⑥



$v(t) = 12 + 8\,e^{-0,75t}\ V, \quad t > 0$

$C = ?$ mF



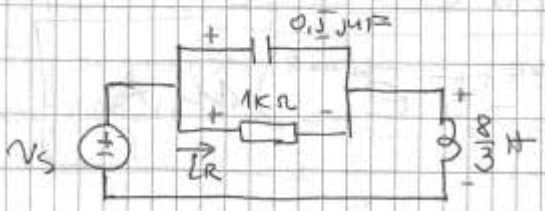
$R_T = 90 + \frac{180 \cdot 40}{180+40} = 122{,}72\ \Omega$

$\tau = R_T C = 122{,}72\ C$

$-0{,}75t = -\frac{t}{\tau} \Rightarrow \tau = \frac{1}{0{,}75} = 1{,}33 = \frac{4}{3}$

$1.33 = 30\,C \Rightarrow \boxed{C = 10{,}8\ \mu F}$

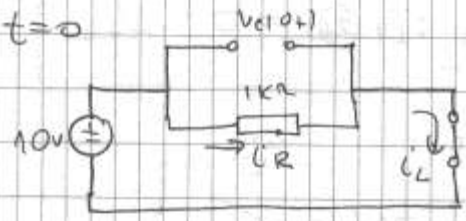
(7)



a) $v_s(t) = 10(-\lambda)$ V

b) $v_s(t) = 10\,\lambda(t)$ V

a) $t=0$



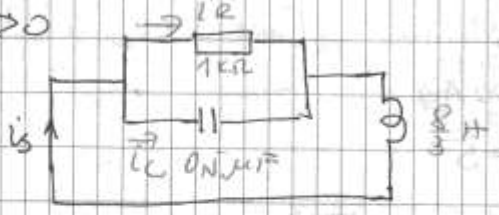
$i_L(0) = \frac{10}{1000} = 0{,}01 = 10\,mA$

$v_c(0+) = 1 \cdot 10^3 \cdot 10\,10^{-3} = 10\,V$

$i_L(0-) = i_L(0+) = 10\,mA$

$v_c(0-) = v_c(0+) = 10\,V$

$t>0$



$-\frac{1}{L}\int v_c\,dt = C\frac{dv_c}{dt} + \frac{v_c}{R}$; $i_s = i_R + i_C$

$\frac{d^2 v_c}{dt^2} + \frac{1}{RC}\frac{dv_c}{dt} + \frac{1}{LC}v_c = 0$

$s^2 + 2000s + 750000 = 0 \Rightarrow s_1 = -500$; $s_2 = -1500$

$v_c(t) = A e^{-500t} + B e^{-1500t}$

$v_c(0+) = 10 = A + B$

$i_L(0+) = i_R(0+) + i_C(0+)$

$v_C'(0+) = \frac{i_L(0+)}{C} - \frac{v_C(0+)}{RC} = 0 \quad v_C'(0+) = 0$

$v_C'(0) = -500A - 1500B = 0$

$A + B = 10$

$\boxed{A = 15 \quad B = -5}$

$\boxed{v_C(t) = 15e^{-500t} - 5e^{-1500t} \text{ V}}$

$\boxed{i_R(t) = (15e^{-500t} - 5e^{-1500t}) \text{ mA}}$

b) $v_s(t) = 10u(t) \text{ V}; \quad v_C(0+) = 0, \; i_L(0+) = 0$

$v_{cf} = 10$

$v_C = 10 + Ae^{-500t} + Be^{-1500t}$

$v_C'(0+) = 2 \times 10^6 [i_L(0+) - i_R(0+)] = 0$

$v_C(0) = 0 = 10 + A + B$

$0 = -500A - 1500B$

$\Rightarrow \quad A = -15; \; B = 5$

$\boxed{v_C(t) = 10 - 15e^{-500t} + 5e^{-1500t} \text{ V}, \; t > 0}$

$\boxed{i_R(t) = 10 - 15e^{-500t} + 5e^{-1500t} \text{ mA}, \; t > 0}$